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Level 3 Mathematics and Statistics (Statistics), 2022

91585 Apply probability concepts in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability concepts in solving problems.	Apply probability concepts, using relational thinking, in solving problems.	Apply probability concepts, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

QUESTION ONE

The IT department of a company keeps track of the devices provided to their employees.
50 randomly selected employees were provided with at least one of the following three devices: a desktop, a smartphone or a tablet. After analysing the survey results, it was found that:

- 22 were provided with a smartphone.
- 34 were provided with a tablet.
- 9 were provided with both a smartphone and a tablet.
- 12 were provided with both a tablet and a desktop.
- 7 were provided with both a smartphone and a desktop
- 5 were provided with all three devices

(a) Calculate the proportion of employees who were provided with a desktop.

(b) Suppose that an employee from this group who was provided with only one device is randomly selected.
What device are they least likely to have been provided with?
Support your answer with statistical reasoning.

(e) The IT department states that 80% of employees have been provided with a tablet. Discuss how carrying out a simulation could determine whether the proportion of the employees provided with a tablet is less than 78%.

You do not need to design the simulation.

ASSESSOR'S USE ONLY

ASSESSOR'S USE ONLY

QUESTION TWO

A workplace requested that their employees (junior and senior level) fill in a voluntary online survey, asking which one of three possible courses (programming, data security or mental health) they would most like to take. Once the closing date for this survey was reached, it was found that 150 employees completed the survey.

62% of these employees were senior employees. 43 senior employees answered that they would like to take a course on mental health, whereas the remaining senior employees opted for data security and programming in the ratio of 3:7.

For junior employees, 25 said they would like to take a course in mental health. The remaining junior employees were equally split between the programming and data security courses.

An employee is chosen at random.

(a) Calculate the probability that the employee is senior and would like to take a course in data security.

(b) Are the events "an employee is junior" and "would like to take a course in programming" mutually exclusive?
Support your answer with appropriate statistical statements.

3

9 1 5 8 6

Level 3 Mathematics and Statistics (Statistics), 2022

91586 Apply probability distributions in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

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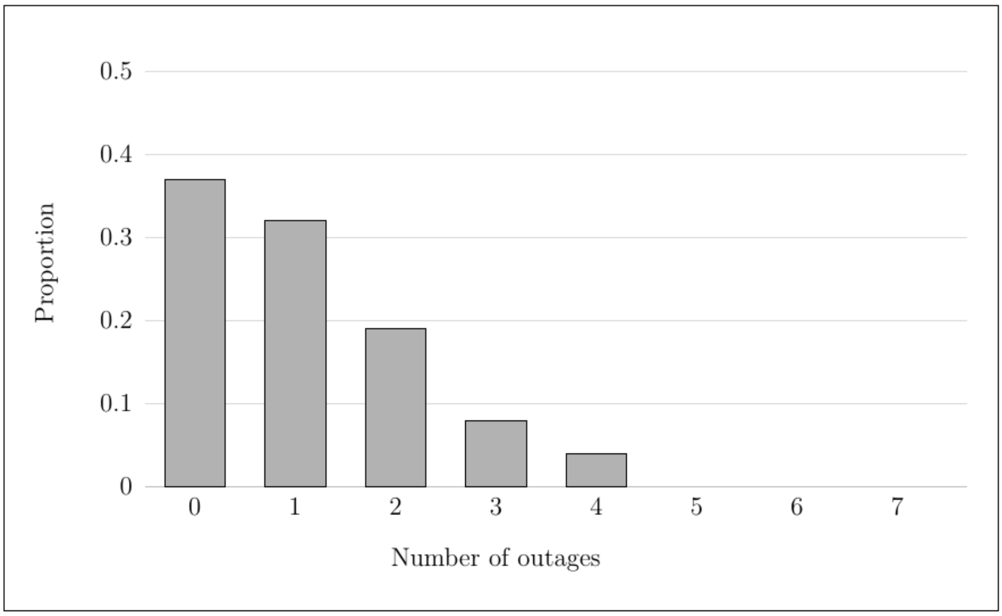
TOTAL

ASSESSOR'S USE ONLY

(b) The internet service provider collects all outage data. The graph below summarises the number of Auckland outages observed in 3-month period. The internet service provider claims that these results could be compared to the Poisson distribution model.



(i) Use the results above to estimate the average number of outages for the observed in 3-month periods.

ASSESSOR'S USE ONLY

(b) The internet provider offers two plans, Broadband and NBN.
Based on the internet service provider's data:

- the download speed for the Broadband plan varies between 25 and 50 Mbps, with the most likely speed being 35 Mbps.
- the download speed for the NBN plan varies between 35 and 80 Mbps, with the most likely speed being 50 Mbps.

(i) On the grid below, sketch the two probability distribution models for the download speed for Broadband and NBN plans.
Clearly label which is Broadband and NBN.

(ii) Calculate an estimate for the probability that at a randomly selected time the download speed of both plans is greater than 35 Mbps but less than 50 Mbps.

QUESTION THREE

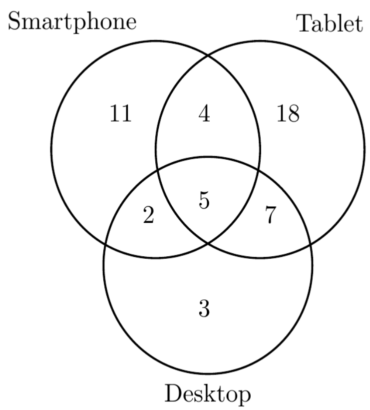
The internet service provider's technical team is often asked to solve various issues. Depending on the complexity of the issues, the time taken to resolve them could vary.

(a) The time taken to resolve hardware issues can be modelled by a normal distribution with a mean of 192 hours and a standard deviation of 48 hours.

(i) Estimate the probability that, given a hardware issue takes longer than 180 hours to resolve, it will take longer than 200 hours to resolve.

(ii) If a hardware issue takes longer than 264 hours to resolve, the internet service provider labels it as a "major issue".
Estimate the probability that three randomly chosen hardware issues are all "major issues".

Assessment Schedule – 2022
Mathematics and Statistics (Statistics): Apply probability concepts in solving problems (91585)
Evidence Statement

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	 $\frac{17}{50} = 0.34$	Correct proportion.		
(b)	$\text{P(only smartphone)} = \frac{11}{32} = 0.344$ $\text{P(only tablet)} = \frac{18}{32} = 0.563$ $\text{P(only desktop)} = \frac{3}{32} = 0.094$ It is least likely that the employee was provided with only a desktop.	One proportion correctly calculated.	Correct conclusion with supporting calculation(s).	

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(e)	The IT department would like to know the percentage of all employees who have been provided with a tablet. A simulation would show the variation in (or distribution of) the number or proportion of employees who were provided with a tablet in samples of size 50, based on the assumption that the proportion is 78%. A comparison of what was observed (34 employees were provided with a tablet or a probability of 0.68) to this simulated distribution can be used to consider the likelihood of the observed result (or a lower proportion) happening, assuming the accuracy rate is 78%.		Discussion of how a simulation would allow one to see that there is variation associated with estimates of proportion, without numerical support given.	A clear discussion of how a simulation would allow one to see that there is variation associated with estimates of proportions, and to make a decision on the proportion, with numerical support given.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

<table>
<tr><th>Q</th><th>Expected Coverage</th><th>Achievement (u)</th><th>Merit (r)</th><th>Excellence (t)</th></tr>
<tr>
<td>(b)</td>
<td>
<table>
<tr><th rowspan="2">Actual position</th><th colspan="3">Predicted position</th><th></th></tr>
<tr><td>Team leader</td><td>Manager</td><td>Executive</td><td></td></tr>
<tr><td>Team leader</td><td>40</td><td>2</td><td>6</td><td>48</td></tr>
<tr><td>Manager</td><td>7</td><td>50</td><td>10</td><td>67</td></tr>
<tr><td>Executive</td><td>2</td><td>5</td><td>28</td><td>35</td></tr>
<tr><td></td><td>49</td><td>57</td><td>44</td><td>150</td></tr>
</table>

P(correct) = (40 + 50 + 28)/150 = 78.7%
P(incorrect) = 21.3%

• 40 / 49 (= 0.816) are correctly predicted as team leaders
• 50 / 57 (= 0.877) are correctly predicted as managers
• 28 / 44 (= 0.636) are correctly predicted as executives

• 40 / 48 (= 0.833) of actual team leaders were correctly predicted
• 50 / 67 (= 0.746) of actual managers were correctly predicted
• 28 / 35 (= 0.8) of actual managers were correctly predicted

The model has a high rate of correct predictions overall, as well as for all three positions individually, with the exception of when the model predicts an executive.

Accept other valid reasoning.
</td>
<td>ONE correct proportion calculated.</td>
<td>At least TWO correct proportions calculated.

AND

Proportions used appropriately to support that the model is good.</td>
<td>States that the model has a high rate of correct prediction with numerical evidence.</td>
</tr>
</table>

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Assessment Schedule – 2022
Mathematics and Statistics (Statistics): Apply probability distributions in solving problems
Evidence Statement

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)(i)	Poisson distribution $\lambda = 1.2$ outages per 3 months $P(X < 3) = 0.3012 + 0.3614 + 0.2169 = 0.8795$	Correct probability calculated.		
(a)(ii)	Poisson distribution $\lambda = 4.8$ outages per year $P(X \geq 2) = 1 - P(X \leq 1) = 1 - (0.0082 + 0.0395) = 0.9523$	Correct adjusted mean calculated.	Correct probability calculated.	
(a)(iii)	• The Poisson distribution assumes that the occurrence of an internet outage is random and unpredictable. This is not the case when outages occur due to planned maintenance, and certain conditions (e.g. extreme weather) will make outages more predictable. • The Poisson distribution assumes that the rate of outages is constant over time. But the number of outages could occur more often at certain times E.g. in winter, where more extreme weather is observed. *Do not accept arguments about simultaneous outages occurring.*	TWO correct reasons in context.	At least ONE correct reason linked to the context. AND Explanation of why it is not appropriate to use a Poisson model.	At least TWO different, correct reasons linked to the context. AND Explanation of why each reason is not appropriate to use a Poisson model.

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(b)	Using μ = 38 hours as the mean for the observed data $z^{-1}{}_{0.38} = 1.175$ $\sigma = \frac{71-38}{1.175} = 28$ which is greater than the standard deviation of the model. Alternative calculation According to the model, $P(X \geq 71) = 8.46\%$, which is less than what the observed data suggests. *Boundedness* – the model includes negative times, which is not realistic as the time taken to resolve software issues would always be positive. Moreover, the normal model assumes an infinite upper limit for the time taken to resolve software issues, where this is not realistic either. In conclusion, the suggested normal distribution is not suitable to model the time taken to resolve software issues.		Suitable calculation to show that the standard deviation of the model is not suitable for the observed data. OR Discussed boundedness as the limitation of the normal model in context.	Suitable calculation to show that the standard deviation of the model is not suitable for the observed data. AND Discussed boundedness as the limitation of the normal model in context. AND Correct conclusion.
(c)	Since the expected number of Type B software issues occurring in a week is smaller than that for Type A, the Type B distribution could have the proportions concentrated at the left end (smllaer *s* values).	Incomplete description.	Valid description of the number of Type B errors occurring in a week with justification.	