3

9 1 5 7 7

Level 3 Calculus, 2022

91577 Apply the algebra of complex numbers in solving problems

Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

QUESTION ONE

(a) Express $\frac{14}{5+3\sqrt{2}}$ in the form $a+b\sqrt{2}$, where a and b are integers.

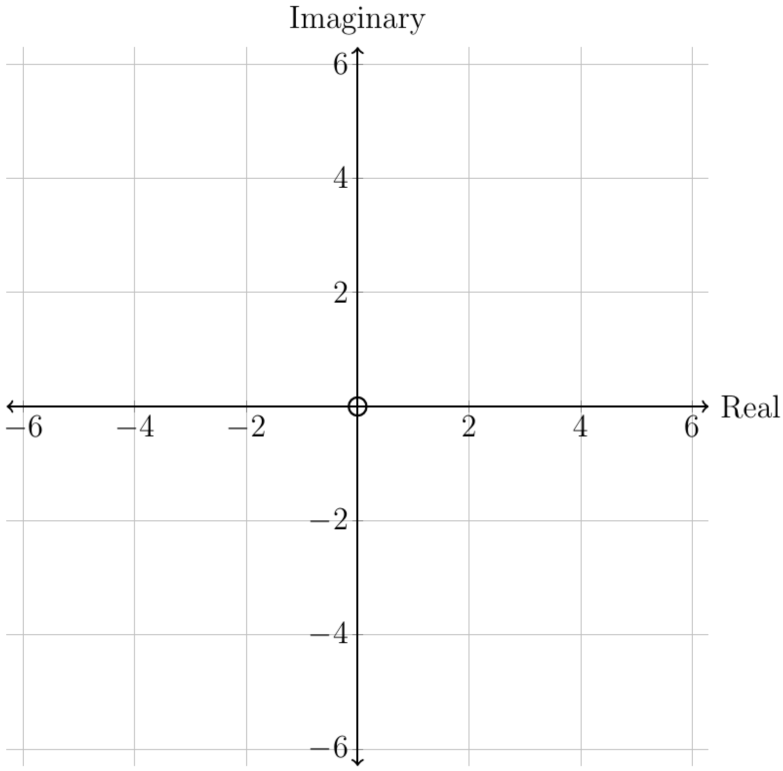
(b) If $(p + q\text{i})(2 - 3\text{i}) = 9 - 7\text{i}$, find the values of p and q.

(c) One solution of $z^3 + Az^2 - z - 14 = 0$ is $z = -2 - \sqrt{3}i$, where A is a real number.
Find the value of A and the other two solutions of the equation.

QUESTION TWO

(a) If $z = -4 + 4\sqrt{3}i$, find $\arg(z)$.

(b) If $z = 4 - 6i$, show $\frac{52}{\overline{z}}$ on the Argand diagram below.



(e) If z is a complex number, where $\mathrm{Im}(z) \neq 0$, and $\dfrac{2z}{1+z^2}$ is a real number, then find $z\overline{z}$.

ASSESSOR'S USE ONLY

3

91578

Level 3 Calculus, 2022

91578 Apply differentiation methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

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TOTAL

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Differentiate $y = (2x - 3x^2)\sin 2x$.

You do not need to simplify your answer.

(b) Find the x-coordinate(s) of any stationary point(s) on the curve with the equation

$$y = 2\ln x - 3x^2, \ x > 0$$

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) A curve is defined by the parametric equations $x = \sqrt{3}\sin 2t$ and $y = \cos^2 t$ where $0 \leq t \leq \pi$.

Find the gradient of the tangent to the curve as a function of t.

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

(c) A curve has the equation $y = \dfrac{\sqrt{x}}{3x+1}$.

Find the equation of the normal to the curve at the point $\left(1, \dfrac{1}{4}\right)$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) Find the x-coordinate(s) of any stationary points on the graph of the function

$$y = e^{2x^2}(2x^2 - 3)$$

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) The tangent to the curve $y = e^x$ at $(1, e)$ is also the tangent to the curve $y = 2\sqrt{x-k}$ at another point.

Find the value of the constant k in terms of e.

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

3

9 1 5 7 9

Level 3 Calculus, 2022

91579 Apply integration methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply integration methods in solving problems.	Apply integration methods, using relational thinking, in solving problems.	Apply integration methods, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Show ALL working.

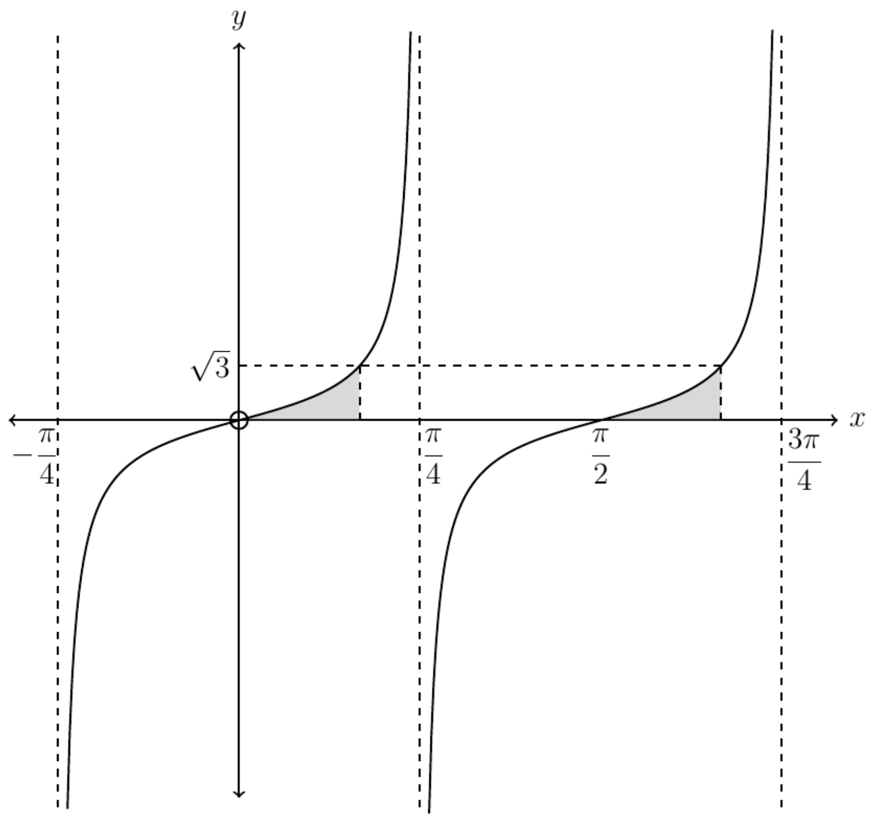
Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

(d) The graph below shows the curve $y = \tan 2x$.

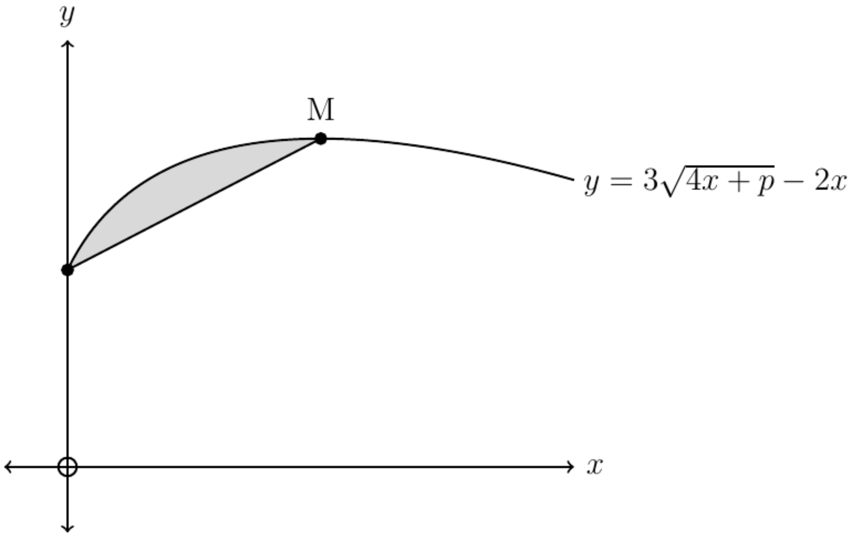


Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

(e) The graph below shows the functions $y = 3\sqrt{4x + p} - 2x$ and $y = qx + 3$, where p and q are constants.

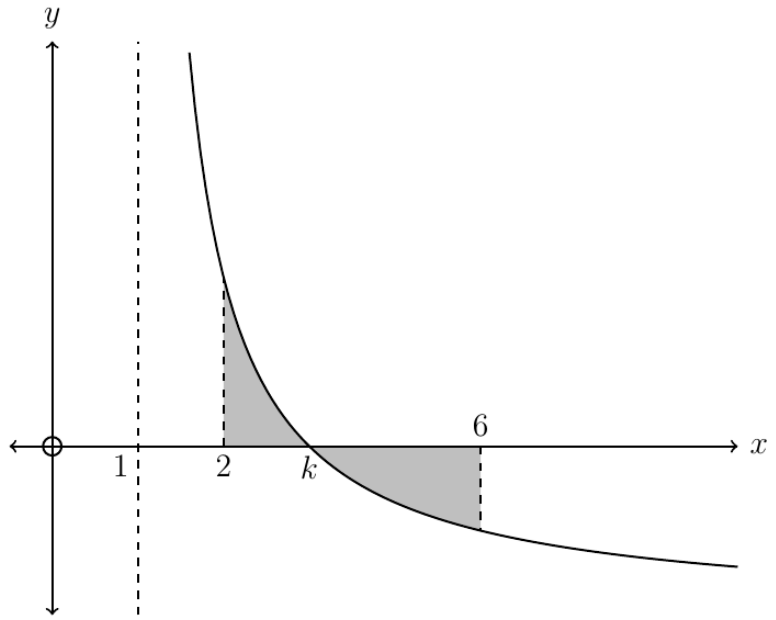
The maximum value of the function $y = 3\sqrt{4x + p} - 2x$ occurs at point M.



Find the values of p and q and the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

(d) The diagram below shows part of the graph of the function $y = \dfrac{x-3}{1-x}$.



Find the shaded area.

You must use calculus and show the results of any integration needed to solve the problem.

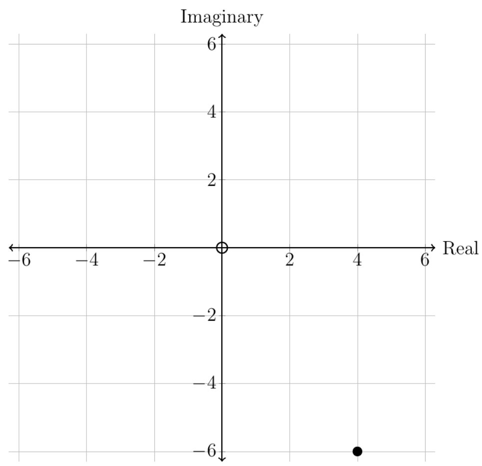
ASSESSOR'S USE ONLY

Assessment Schedule – 2022

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$\frac{14}{5+3\sqrt{2}} = \frac{14}{5+3\sqrt{2}} \times \frac{5-3\sqrt{2}}{5-3\sqrt{2}}$ $= \frac{14\left(5-3\sqrt{2}\right)}{25-18}$ $= 10-6\sqrt{2}$	Correct answer.		
(b)	$(p+qi)(2-3i) = 9-7i$ $2p+3q+(-3p+2q)i = 9-7i$ Comparing the real parts and the imaginary parts gives the simultaneous equations $2p+3q=9$ and $-3p+2q=-7$. Solving these gives $p = 3$ and $q = 1$.	Simplified LHS correctly.	Correct values of p and q.	
(c)	By the conjugate root theorem $z=-2+\sqrt{3}i$ is also a solution. $\left(z-\left(-2+\sqrt{3}i\right)\right)\left(z-\left(-2-\sqrt{3}i\right)\right)(z-k)=0$ $\left(z+2-\sqrt{3}i\right)\left(z+2+\sqrt{3}i\right)(z-k)=0$ $(z^2+4z+7)(z-k)=0$ $z^3+4z^2+7z-kz^2-4kz-7k=0$ $z^3+(4-k)z^2+(7-4k)z-7k=0$ $k = 2$. Therefore $A = 2$ and the other two solutions are $z=2$ and $z=-2+\sqrt{3}i$	Stated that $z=-2+\sqrt{3}i$ is another solution.	Found the value of A and the other two solutions with correct steps.	

	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\arg(z) = \pi - \tan^{-1}\left(\frac{4\sqrt{3}}{4}\right)$ $= \pi - \frac{\pi}{3}$ $= \frac{2\pi}{3}$	Correct answer.		
(b)	$\frac{52}{\bar{z}} = \frac{52}{4+6i} \times \frac{4-6i}{4-6i}$ $= \frac{52(4-6i)}{52}$ $= 4-6i$ 	Correct solution plotted on Argand diagram.		
(c)	$\sqrt{x-w} = \sqrt{x+w} - 4$ $\sqrt{x-w}^2 = \left(\sqrt{x+w} - 4\right)^2$ $x - w = x + w - 8\sqrt{x+w} + 16$ $4\sqrt{x+w} = w + 8$ $\left(4\sqrt{x+w}\right)^2 = (w+8)^2$ $16x + 16w = w^2 + 16w + 64$ $x = \frac{w^2 + 64}{16}$	Correct squaring of both sides.	Correct answer.	

Assessment Schedule – 2022
Calculus: Apply differentiation methods in solving problems (91578)
Evidence Statement

	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$\frac{dy}{dx} = (2-6x)\times\sin 2x + (2x-3x^2)\times\cos 2x \times 2$	Correct derivative.		
(b)	$y = 2\ln x - 3x^2$ $\frac{dy}{dx} = \frac{2}{x} - 6x = \frac{2-6x^2}{x}$ Solving $\frac{dy}{dx} = 0$ gives $\frac{2-6x^2}{x} = 0$ $1-3x^2 = 0$ $x = \frac{1}{\sqrt{3}}$	Correct derivative.	Correct solution with correct derivative.	
(c)	$\frac{dx}{dt} = 2\sqrt{3}\cos 2t$ $\frac{dy}{dt} = -2\cos t \sin t$ $\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx}$ $= \frac{-2\cos t\sin t}{2\sqrt{3}\cos 2t}$ $= -\frac{\sin 2t}{2\sqrt{3}\cos 2t}$ $= -\frac{1}{2\sqrt{3}}\tan 2t$ ● Accept any of the last three lines.	Correct $\frac{dx}{dt}$ OR Correct $\frac{dy}{dt}$.	Correct solution with correct derivatives shown.	

	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(d)	$v = \frac{ds}{dt} = e^{t-8} - e^{-t+2}$ Solving $\frac{ds}{dt} = 0$ gives $e^{-t+2}(e^{2t-10} - 1) = 0$ $e^{2t-10} = 1$ $2t - 10 = 0$ $t = 5$ Since $\frac{d^2s}{dt^2} = e^{5-8} + e^{-5+2} > 0$ the minimum velocity occurs when $t = 5$.	Correct derivative.	Correct answer with correct derivative and justification.	
(e)	Note that r does not change as water drains out. $V = \frac{1}{3}\pi\left((h+r)^2 \times (h+r) - r^2 \times r\right)$ $= \frac{1}{3}\pi\left((h+r)^3 - r^3\right)$ Differentiating $\frac{dV}{dh} = \pi(h+r)^2$ Moreover $\frac{dV}{dt} = \pi r^2 \times \left(-c\sqrt{5h}\right)$ $= -\pi r^2 c\sqrt{5h}$ Therefore $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $= \frac{1}{\pi(h+r)^2} \times \pi r^2 c\sqrt{5h}$ $= \frac{r^2 c\sqrt{5h}}{(h+r)^2}$ m per second	Correct $\frac{dV}{dh}$ or $\frac{dV}{dt}$.	Stated the relationship $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$.	T1: All steps are correct but contains a minor algebraic error. T2: Correct solution with correct derivatives.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	T1	T2

Assessment Schedule – 2022

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$3x-2\sqrt{x}+c$	Correct solution.		
(b)	$\int_0^1 (ax+1)\ dx=0$ $\left[\frac{a}{2}x^2+x\right]_0^1=0$ $\frac{a}{2}+1=0$ $a=-2$	Correct solution.		
(c)	$\frac{dy}{dx}=\frac{\sec^2 x}{\tan x}$ $\int dy=\int\frac{\sec^2 x}{\tan x}dx$ $y=\ln(\tan x)+c$ Since $y=2$ when $x=\frac{\pi}{4}$ $2=\ln\left(\tan\frac{\pi}{4}\right)+c$ $2=\ln(1)+c$ $2=c$ Therefore $y=\ln\left(\tan\frac{\pi}{3}\right)+2=\ln\left(\sqrt{3}\right)+2$	Correct integration.	Correct solution with correct integration.	

	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(e)	Let k be a negative constant. $\frac{dT}{dt} = k(T-25)$ $\int \frac{1}{T-25}\, dT = \int k\ dt$ $\ln(T-25) = kt + c$ $T - 25 = e^{kt+c}$ $T = Ae^{kt} + 25$ Since $T = 105$ when $t = 0$, $A = 80$. We can write $T = 80e^{kt} + 25$. Since $T = 90$ when $t = 1$ $80e^{k} + 25 = 90$ $e^{k} = \frac{65}{80}$ $k = \ln\left(\frac{13}{16}\right)$ Therefore $T = 80e^{\ln\left(\frac{13}{16}\right)\times 1.5} + 25 = 83.59\ ^\circ\text{C}$	Correct general solution to DE. $+\,c$ is not required.	Correct general solution to DE plus correct values of A and k.	Correct solution (units not required)

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t