2

9 1 2 6 1

Level 2 Mathematics and Statistics, 2022

91261 Apply algebraic methods in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Simplify $\dfrac{3x-15}{x^2+2x-35}$.

(b) Express $x^2 + 16x + 35$ in completed square form, i.e. $(x + a)^2 + b$, where a and b are integers.

(c) The quadratic equation $x^2 + ax + b = 0$ has solutions $x = -3$ and $x = 2$.
Solve the equation $x^2 + bx + a = 0$.

ASSESSOR'S USE ONLY

(c) Solve the equation $7^{4n+2} + 7^{4n-1} = 7^{3n+2} + 7^{3n-2}$.

(d) When the price of a monitor is increased by x%, the number sold decreases by $0.5x$%.
Determine how much the price should be increased by in order to increase the total revenue by 12%.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Simplify $\log(36) - \log(6)$.

(b) Solve $\log(3) + \log(x^2 + 2) = \log(153)$.

(c) Lisa borrows \$125,000 from her bank to start a business at the start of 2022.
Interest is compounded on the principal at the end of each year.
The total amount of Lisa's loan after n years is given by

$$A = 125000\left(1 + \frac{r}{100}\right)^n$$

where r% is the compound interest rate per year.
By the end of 2024, the value of Lisa's loan will be \$175,616.
Show that $r = 12$.

2

9 1 2 6 2

Level 2 Mathematics and Statistics, 2022

91262 Apply calculus methods in solving problems

Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You must show the use of calculus in answering all questions in this paper.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

QUESTION ONE

ASSESSOR'S USE ONLY

(a) A function f is given by $f(x) = 4x^3 + 3x^2 + 7x + 1$.
Find the gradient of the graph of the function at the point where $x = -1$.

(b) At t seconds, the acceleration of an object in ms^{-2} is given by $a(t) = 6t - 18$.
The object is initially at the origin O, moving with a velocity of 24 ms^{-1}.

(i) What is the speed of the object at $t = 3$ seconds?

(ii) What is the distance of the object from O when it changes direction for the first time?

(c) The volume of water inside a water trough, in cm^3, is given by $V = 24h^2 + 4800h$, where h is the depth of water in centimetres.

ASSESSOR'S USE ONLY

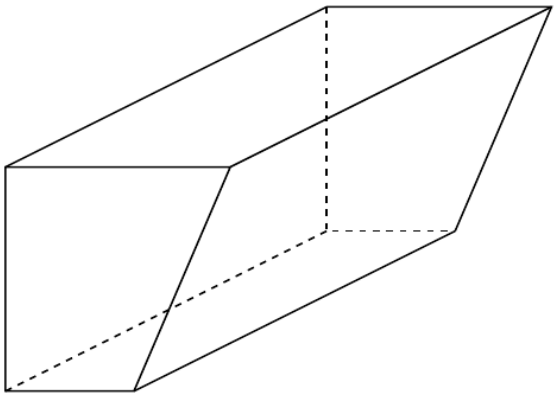
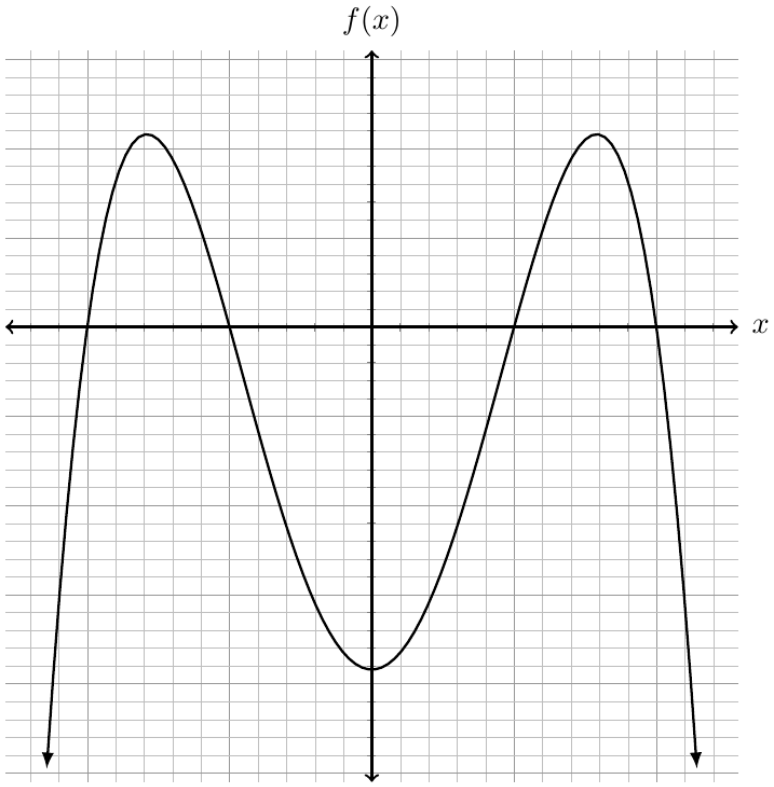


Diagram is NOT to scale

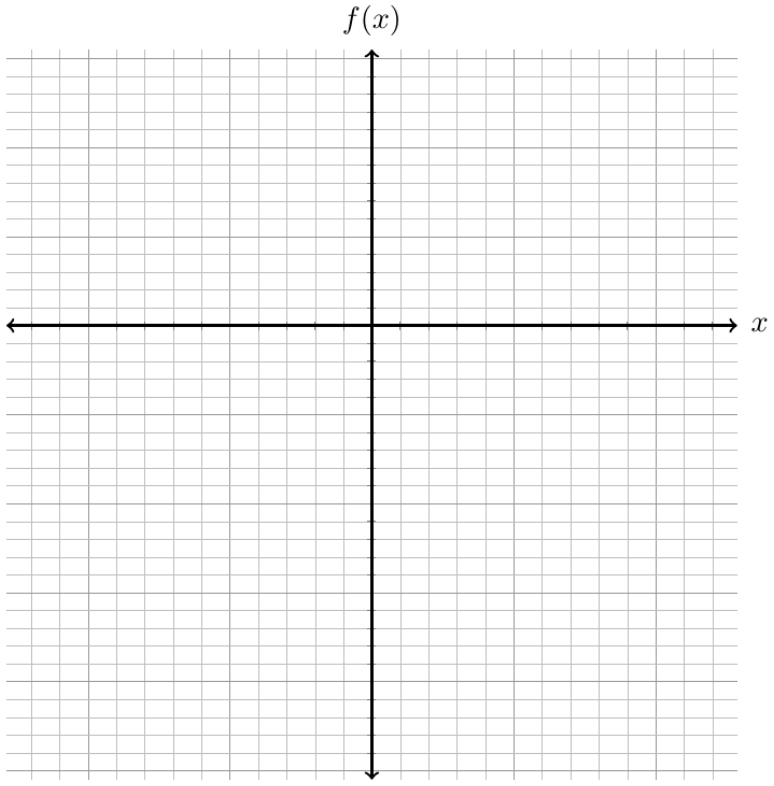
Water is poured into the water trough, which is initially empty, such that the depth of water is increasing at a constant rate of 0.4 cm per second.
Find the depth of water at the instant when the rate of change of volume is 2,112 cm^3 per second.

(d) The graph of a function $y = f(x)$ is shown on the axes below.

ASSESSOR'S USE ONLY



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.
Both sets of axes have the same horizontal scale.



2

9 1 2 6 7

Level 2 Mathematics and Statistics, 2022

91267 Apply probability methods in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

QUESTION ONE

From March 2022, New Zealand resumed international travel. A large number of people have been travelling overseas.

(a) In order to get more insight, a random sample of 550 people who went overseas in 2022 were surveyed on their destination and the purpose of their trip.
The following table shows the results.

Table 1: Overseas trip and purpose 2022

	Holiday	Work	Migration	Total
Australia	100	112	44	256
Europe	70	22	32	124
South-East Asia	150	16	4	170
Total	320	150	80	550

(i) According to the results of this survey, what proportion of people travelled to Europe?

(ii) What proportion of people that migrated went to South-East Asia?

(iii) Is it more likely that a person who travelled to Australia went for work or that a person who travelled to Europe went for work?
Include appropriate calculations to support your answer.

ASSESSOR'S USE ONLY

(b) The airline also has been flying from New Zealand to Tokyo, Japan.
For Tokyo bound flights, the longest 10% of travel times lie above 810.48 minutes and the shortest 15% of travel times lie below 683 minutes.

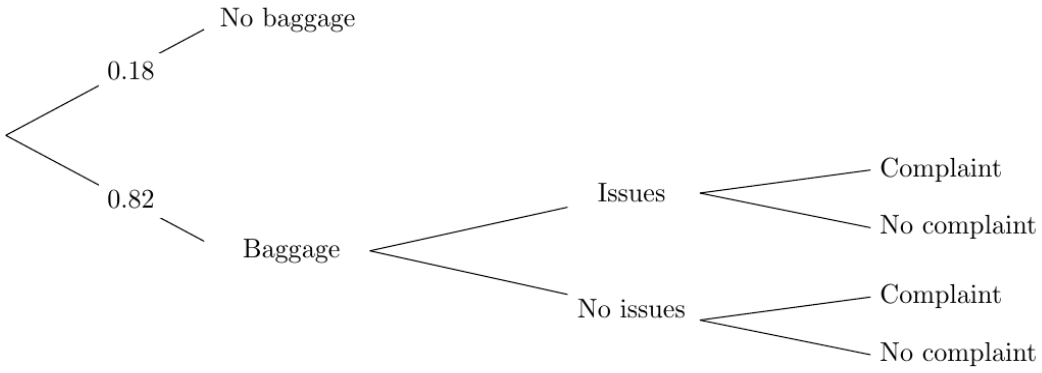
Assume that a normal distribution can be used to model the flight time for Tokyo bound flights.

Calculate the mean and standard deviation of flight times for Tokyo bound flights.

ASSESSOR'S USE ONLY

(b) 82% of the customers who arrive at Christchurch Airport have at least one piece of baggage to claim. The airport states that there are baggage claim issues in 2 in 25 customers. In addition, if there is an issue, it is four times as likely for the customer to lodge a complaint as not lodging a complaint. If there are no issues, 95% of customers do not lodge a complaint.

Determine how many times more likely it is for a random customer to not lodge a baggage complaint than to lodge one.



Assessment Schedule – 2022
Mathematics and Statistics: Apply algebraic methods in solving problems (91261)
Evidence Statement

Q ONE	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\dfrac{3x-15}{x^2+2x-35}$ $=\dfrac{3(x-5)}{(x+7)(x-5)}$ $=\dfrac{3}{x+7}$	Correct answer.		
(b)	$x^2+16x+35$ $=x^2+16x+64-29$ $=(x+8)^2-29$	Correct answer.		
(c)	The given quadratic equation is $(x+3)(x-2)=0$ $x^2+x-6=0$ The quadratic equation to be solved is $x^2-6x+1=0$ $x=\dfrac{6\pm\sqrt{32}}{2}$ $=3\pm2\sqrt{2}$ x = 0.17, 5.83 or equivalent.	Correct quadratic equation found.	Correct solutions.	
(d)	$D=4^2-4\times(-3)\times(-3m+2)$ $=16-36m+24$ $=4(10-9m)$ Solving $D<0$ gives $m>\dfrac{10}{9}$.	Sets discriminant less than 0.	Correct inequality for m obtained.	

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(e)	Let $a = x - y$. $(8-2x+2y)(x-y-8) > -40$ $(x-y-4)(x-y-8) < 20$ $(a-4)(a-8) < 20$ $a^2 - 12a + 12 < 0$ $a^2 - 12a + 36 - 24 < 0$ $(a-6)^2 - 24 < 0$ $\left(a-6-\sqrt{24}\right)\left(a-6+\sqrt{24}\right) < 0$ $\left(x-y-6-2\sqrt{6}\right)\left(x-y-6+2\sqrt{6}\right) < 0$ Solving the last inequality gives $6-2\sqrt{6} < x-y < 6+2\sqrt{6}$ But $2 < x-y$, and $6-2\sqrt{6} < 2$ so the final inequality is $2 < x-y < 6+2\sqrt{6}$. Hence the upper bound on $x-y$ is $6+2\sqrt{6}$. ● Also accept $x-y < 10.899$.	At least one step taken to correctly simplify the given inequality.	Completed the square of the LHS expression to obtain line 5 or line 6.	T1: Correct inequality on $x-y$ found with correct steps. T2: Correct upper bound.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Part Q correct.	1 of u	2 of u	3 of u	1 of r	2 of r	T1	T2

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(d)	$\frac{3x+1}{x-1} = -\frac{x+4}{x-2}$ $(3x+1)(x-2) = -(x+4)(x-1)$ $3x^2 - 6x + x - 2 = -x^2 + x - 4x + 4$ $4x^2 - 2x - 6 = 0$ $2x^2 - x - 3 = 0$ $(2x-3)(x+1) = 0$ $x = -1,\ \frac{3}{2}$ The points of intersection are $(-1,1)$ and $\left(\frac{3}{2},11\right)$.	Correct equation formed.	Correct points of intersection.	
(e)	$8(x+2)^2 - 8k^2 = 16k + 8$ $(x+2)^2 - k^2 = 2k + 1$ $x^2 + 4x + 4 - k^2 - 2k - 1 = 0$ $D = 4^2 - 4 \times 1 \times (4 - k^2 - 2k - 1)$ $= 4(4 - 4 + k^2 + 2k + 1)$ $= 4(k^2 + 2k + 1)$ $= 4(k+1)^2$ This means that any value of k satisfies $D \geq 0$. The restrictions $2k > 0$ and $16k + 8 > 0$ give $k > 0$. Solving the quadratic equation using the quadratic formula $x = \frac{-4 \pm \sqrt{4(k+1)^2}}{2 \times 1} = -2 \pm (k+1)$. So $x = -1 + k$ (negative result ignored). Since $x + 2 > 2k$, solving $x + 2 = k + 1 > 2k$ gives $k < 1$. Therefore, the final restriction on k is $0 < k < 1$.	Correct equation formed.	Consistently calculated the discriminant in terms of k.	T1: Stated $k > 0$ as the final answer with all necessary mathematical justification. T2: Found the correct final inequality $0 < k < 1$.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Part Q correct.	1 of u	2 of u	3 of u	1 of r	2 of r	T1	T2

Assessment Schedule – 2022
Mathematics and Statistics: Apply calculus methods in solving problems (91262)
Evidence Statement

Q ONE	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$f'(x) = 12x^2 + 6x + 7$ $f'(-1) = 12(-1)^2 + 6(-1) + 7$ $= 12 - 6 + 7$ $= 13$	Gradient found.		
(b)(i)	$v(t) = 3t^2 - 18t + c$ Since $v(0) = 24$, $c = 24$ Therefore, the velocity of the object when $t = 3$ is $v(3) = 3(3)^2 - 18(3) + 24$ $= -3$ ms^{-1} The speed of the object is 3 ms^{-1}.	Correct velocity function found. OR Stated that the speed of the object is –3 ms^{-1}.	Correct answer.	
(b)(ii)	$v(t) = 3t^2 - 18t + 24$ $= 3(t^2 - 6t + 8)$ $= 3(t - 2)(t - 4)$ The object changes direction for the first time at $t = 2$. $s(t) = t^3 - 9t^2 + 24t + c$ Since $s(0) = 0$, $c = 0$ $s(2) = 2^3 - 9(2)^2 + 24(2)$ $= 20$ m Therefore, the object is 20 m from O.	Correctly found the first time when the objection changes direction.	Correct distance from the origin.	

<table>
<tr><th>Q</th><th>Expected Coverage</th><th>Achievement (u)</th><th>Merit (r)</th><th>Excellence (t)</th></tr>
<tr>
<td>(d)</td>
<td>
$f'(x) = -4x^3 + 16a^2x$
$= -4x(x^2 - 4a^2)$
$= -4x(x-2a)(x+2a)$

The function has stationary points at $x = -2a$, $x = 0$ and $x = 2a$.

To identify the nature of these stationary points, substitute the values in between the stationary points.

$f'(-3a) = -4(-3a)(-3a-2a)(-3a+2a) = 60a^3 > 0$
$f'(-a) = -4(-a)(-a-2a)(-a+2a) = -12a^3 < 0$
$f'(a) = -4(a)(a-2a)(a+2a) = 12a^3 > 0$
$f'(3a) = -4(3a)(3a-2a)(3a+2a) = -60a^3 < 0$

OR in a table:
<table>
<tr><td>x</td><td>…</td><td>$-2a$</td><td>…</td><td>0</td><td>…</td><td>$2a$</td><td>…</td></tr>
<tr><td>$f'(x)$</td><td>+</td><td>0</td><td>–</td><td>0</td><td>+</td><td>0</td><td>–</td></tr>
<tr><td>$f(x)$</td><td>↗</td><td>max</td><td>↘</td><td>min</td><td>↗</td><td>max</td><td>↘</td></tr>
</table>
Since $x = 2 - 2b < 0$ and $x = b > 0$ we obtain $2 - 2b = -2a$ and $b = 2a$.

Solving these equations simultaneously gives $a = 1$ and $b = 2$.
</td>
<td>Correct derivative found.</td>
<td>x-coordinates of stationary points found.</td>
<td>T1: Correct values of a and b found showing all necessary mathematical justification.

T2: Correct values of a and b found showing all necessary mathematical justification.

AND

Shown that both turning points are maxima.</td>
</tr>
</table>

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Part Q correct.	1 of u	2 of u	3 of u	1 of r	2 of r	T1	T2

Assessment Schedule – 2022
Mathematics and Statistics: Apply probability methods in solving problems (91267)
Evidence Statement

<table>
<tr><th>Q
ONE</th><th>Expected Coverage</th><th>Achievement (u)</th><th>Merit (r)</th><th>Excellence (t)</th></tr>
<tr><td>(a) (i)</td><td>$\text{P(Europe)} = \frac{124}{550} = 0.2255$</td><td>Proportion found.</td><td></td><td></td></tr>
<tr><td>(a) (ii)</td><td>$\text{P(SE Asia if migration)} = \frac{4}{80} = 0.05$</td><td>Proportion found.</td><td></td><td></td></tr>
<tr><td>(a) (iii)</td><td>$\text{P}\left(\text{Work if travelled to Aus}\right)$
$= \frac{112}{256} = 0.4375$

$\text{P}\left(\text{Work if travelled to Europe}\right)$
$= \frac{22}{124} = 0.1774$

It is more likely that a person who travelled to Australia went for work.</td><td>One correct conditional probability.</td><td>Two correct conditional probabilities compared.</td><td></td></tr>
<tr><td>(b) (i)

(b) (ii)</td><td><table><tr><th></th><th>Solo</th><th>Group</th></tr><tr><td>Australia</td><td>72</td><td>28</td></tr><tr><td>Europe</td><td>15</td><td>55</td></tr><tr><td>South-East Asia</td><td>85</td><td>65</td></tr><tr><td>Total</td><td>172</td><td>148</td></tr></table>
$\frac{172+150}{550} \times n = 346$
$n = 591$</td><td>Correct number of people in the new group.

OR

Errors in table but consistent final answer.</td><td>Correct numbers in the table.

AND

Correct number of people in the new group.</td><td></td></tr>
</table>

	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a) (iv)	**Centre**: *Compares means or medians* Sample data median (1538) is greater than normal model (1500) so the model does not fit the data. A different normal model with a mean of 1538 might fit better. OR A normal distribution is symmetric (mean = median), whereas the sample data is skewed to the right (mean > median). Therefore, this data does not seem to be normally distributed. **Spread**: *Compares IQR, Range, or σ* The IQR of the data is slightly greater than the model, suggesting the data is more spread out than model. The expected range would be ±240 (3 × 80) from the mean so from 1260 to 1740, but the data covers 1445 to 1650 (indicating its standard deviation would be about 205/6 = 34 rather than 80). **Shape**: The data is not symmetrical nor unimodal, which would be expected for a normal distribution. It is skewed to the right (higher chance of really long flight times than what the model suggests) and has several peaks (somewhere in intervals 1520 - 1530, 1580 - 1590, and 1625 – 1635), indicating that the sample data is bi/trimodal (flights with the same flight times).	Two valid comments about different aspects of shape, centre, spread or comment on the quality of the testing.	Two valid comments comparing centre, spread or shape of the data to the model.	At least two different valid comments comparing centre, spread, or shape of the data to the expected model, with context or evidence explaining why the normal model is or isn't appropriate to this context.

	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(b)	$P(Y > 810.48) = 0.1$ $P\left(0 < z < \frac{810.48 - \mu}{\sigma}\right) = 0.4$ $\frac{810.48 - \mu}{\sigma} = 1.281$ $P(Y < 683) = 0.15$ By symmetry, $P(Y > 2\mu - 683) = 0.15$ So $P\left(0 < z < \frac{\mu - 683}{\sigma}\right) = 0.35$ $\frac{\mu - 683}{\sigma} = 1.036$ Solving these two equations simultaneously gives μ = 740 minutes and σ = 55 minutes. ● Accept reasonable rounding errors.	CAO. OR Correct z-values found (±1.281 or ±1.036).	Correct mean found. OR correct standard deviation found.	Correct mean and standard deviation are found (with units), with clear working.

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	A valid attempt at one question.	1 of u	2 of u	3 of u	1 of r	2 of r	T1	T2