2

9 1 2 6 1

Level 2 Mathematics and Statistics, 2021 v1

91261 Apply algebraic methods in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. Guess-and-check methods, and correct answer(s) only, will generally limit grades to Achievement.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

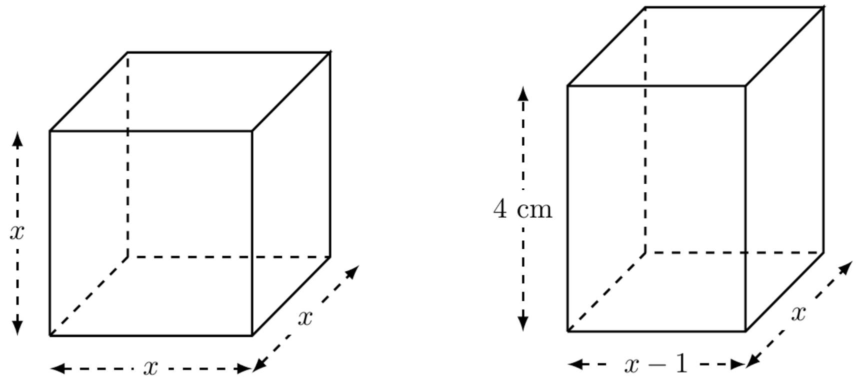
QUESTION ONE

(a) Write as a single logarithm $\log_3(x)+\log_3(6x)-\log_3(x^2)$.

(b) Find the value of k if $\log_4(k)=3$.

(c) Find an expression for n in terms of m for the equation $\frac{8^{4m}}{8^{m-1}}=\frac{64^{m+2}}{8^n}$.

(e) Julia buys a square box and a rectangular box for packing her clothes.
The combined surface area of the two boxes, including their bases, is 106 cm^2.



Find the volume of the square box.

ASSESSOR'S USE ONLY

QUESTION THREE

(a) Find the discriminant of the quadratic equation $x^2 + 6x = 2$.

(b) The quadratic equation $kx^2 - 11x + r = 0$ has the solutions $-\frac{2}{3}$ and $\frac{5}{2}$.

Find the values of k and r.

(c) Write $\frac{y}{y^2 - 49} - \frac{y+2}{y^2 - 5y - 14}$ as a single fraction in its simplest form.

2

9 1 2 6 2

Level 2 Mathematics and Statistics, 2021 v1

91262 Apply calculus methods in solving problems

Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You must show the use of calculus in answering all questions in this paper.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

QUESTION ONE

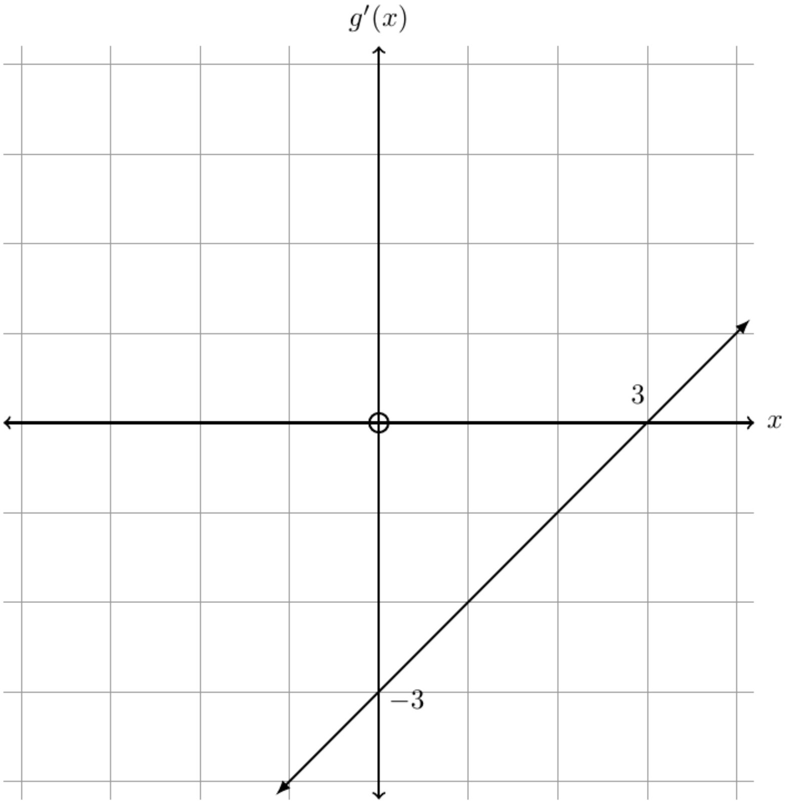
ASSESSOR'S USE ONLY

(a) A function f is given by $f(x) = -x^3 + 3x - 7$.
Find the gradient of the graph of the function at the point where $x = 3$.

(b) Find the coordinates of the point(s) on the curve $y = x^3 + 3x^2 - 9x + 2$ where the tangent to the curve is parallel to the x-axis.

(c) Find the value(s) of the constant k for which the graph of the function $f(x) = -\frac{x^3}{3} - 2x^2 + kx + 2$ is always decreasing.

(b) The graph of a function $y = g'(x)$ is shown on the axes below.



It is given that $g(2) = 0$.

Find an expression for $g(x)$.

You must use calculus to obtain your answer.

(d) The volume, V cm^3, of an expanding square pyramid is given by

$$V = 6x^3$$

where x is the side length of the base of the pyramid.

Find the side length of the pyramid when the rate of change of the volume of the pyramid with respect to the side length is 36 cm^3 / cm.

(e) The tangent to the curve $y = x^2 + 1$ at the point $(-2, 5)$ is also a tangent to the curve $y = x^3 - kx - 1$, where k is a constant.

Use calculus to find the value of k.

ASSESSOR'S USE ONLY

2

9 1 2 6 7

Level 2 Mathematics and Statistics, 2021 v1

91267 Apply probability methods in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

QUESTION ONE

Mike runs a local fruit and vegetable shop.
Cauliflowers are popular among his customers.

https://www.odt.co.nz/news/national/10-cauliflower-prices-skyrocket-wet-weather

(a) The weights of the cauliflowers can be modelled by a normal distribution with a mean of 628 g and a standard deviation of 160 g.

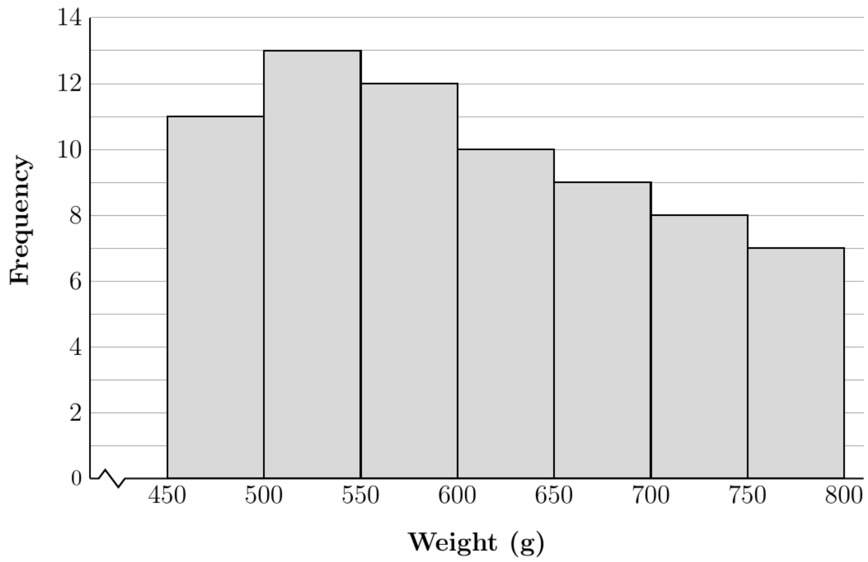
(i) Find the range of weights in which the middle 95% of cauliflowers lie.

(ii) Find the probability that a randomly chosen cauliflower from Mike's shop weighs between 536 g and 720 g.

(b) Red cabbages are also popular at Mike's shop.
Mike's supplier claims that the weights of the red cabbages in the shop can be modelled by a normal distribution with a mean of 590 g and a standard deviation of 62 g.

Mike wanted to verify his supplier's claim. He randomly chose 70 red cabbages from his shop and weighed each of them. The results are shown in Figure 1 below.

Figure 1: Weights of Red Cabbages



Compare the shape, centre, and spread of the histogram in Figure 1 with the claimed normal distribution.
You should provide numerical evidence where appropriate.

ASSESSOR'S USE ONLY

(ii) In reality, the probability of employees wanting to continue their employment depends on the type of their employment and whether or not they are students.

For a casual employee, the probability that the employee wants to continue their employment is greater by 0.2 for students compared to non-students.
The probability that a casual employee wants to continue their employment is 0.47.

Find, for casual employees, how many times as likely a student does not want to continue their employment, compared to an employee who is not a student not wanting to continue their employment.

ASSESSOR'S USE ONLY

(b) There is a competing juice stall several blocks away.
The following table shows how the competitor performed each day by season last year.

Table 2: Performance of competitor's juice stall last year

Season	Made a profit	Made a loss	Broke even	Total
Spring	62	23	5	90
Summer	74	9	8	91
Autumn	25	35	29	89
Winter	5	45	40	90
Total	166	112	82	360

For each of Mike's and his competitor's juice stalls, calculate for each stall how many times as likely the stall is to make a profit in summer compared to spring.
Comment on the validity of your calculations.

Assessment Schedule – 2021 v1
Mathematics and Statistics: Apply algebraic methods in solving problems (91261)
Evidence Statement

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$\log_3(x)+\log_3(6x)-\log_3(x^2)$ $=\log_3\left(\frac{6x^2}{x^2}\right)$ $=\log_3(6)$	Correct answer.		
(b)	$\log_4(k)=3$ $k=4^3$ $=64$	Correct answer.		
(c)	$\frac{8^{4m}}{8^{m-1}}=\frac{64^{m+2}}{8^n}$ $8^{4m-(m-1)}=8^{2(m+2)-n}$ $4m-m+1=2m+4-n$ $n=-m+3$	Changed base to 8.	Correct expression.	
(d)	Using the quadratic formula, the solutions of the equation $ax^2+bx+c=0$ are $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Similarly, the solutions of the equation $cx^2+2bx+4a=0$ are $x=\frac{-2b\pm\sqrt{(2b)^2-16ac}}{2c}$ $=\frac{-2b\pm\sqrt{4b^2-16ac}}{2c}$ $=\frac{-b\pm\sqrt{b^2-4ac}}{c}$ Therefore, the solutions of the first equation are equivalent to $\frac{2a}{c}$ times the solutions of the second equation.	Solutions to one equation found.	Solutions to both equations found.	Correct conclusion reached.

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$(6p^{-1}q^{0.5})^{-2} = \left(\frac{6q^{0.5}}{p}\right)^{-2}$ $= \left(\frac{p}{6q^{0.5}}\right)^{2}$ $= \frac{p^2}{36q}$	Correct answer.		
(b)	$2ac + 3bd + 3ad + 2bc$ $= 2c(a+b) + 3d(b+a)$ $= (2c+3d)(a+b)$	Factors by either $(a + b)$ or $(2c + 3d)$.	Correct final expression.	
(c)	$y^2 + 4xy = 5x^2$ $y^2 + 4xy + 4x^2 = 5x^2 + 4x^2$ $(y+2x)^2 = 9x^2$ $y + 2x = \pm 3x$ $y = -2x \pm 3x$ $= -5x,\ x$ ● Accept using the quadratic formula to find the same results.	Expressed the LHS of the equation as $(y + 2x)^2$.	Both solutions are found.	
(d)	$625^{\frac{3x-4p}{4}} = 5^{px^2}$ $5^{4\times\frac{3x-4p}{4}} = 5^{px^2}$ $3x - 4p = px^2$ $px^2 - 3x + 4p = 0$ The discriminant is $\Delta = (-3)^2 - 4 \times p \times 4p$ $= 9 - 16p^2$ $= (3-4p)(3+4p)$ Therefore, the two exponentials do not intersect when $p < -\frac{3}{4}$ or $p > \frac{3}{4}$.	Changes base to 5 with the correct exponents.	Correct discriminant found.	Correct restrictions found.

Assessment Schedule – 2021 v1

Mathematics and Statistics: Apply calculus methods in solving problems (91262)

Evidence Statement

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$f'(x) = -3x^2 + 3$ $f'(3) = -3 \times 9 + 3$ $= -24$	Gradient found.		
(b)	$\frac{dy}{dx} = 3x^2 + 6x - 9$ $= 3(x^2 + 2x - 3)$ $= 3(x+3)(x-1)$ Since the gradient of all lines parallel to the x-axis is 0 $3(x+3)(x-1) = 0$ $x = -3,\ 1$ $(-3, 29)$ and $(1, -3)$	Correct answer.		
(c)	$f'(x) = -x^2 - 4x + k$ $= -(x^2 + 4x + 4) + 4 + k$ $= -(x+2)^2 + 4 + k$ Setting $f'(x) < 0$ gives $k < -4$ for all x.	Correct inequality $f'(x) < 0$.	Correct restriction on k.	
(d)	$\frac{dy}{dx} = 6x^2 + 8x$ At the point $(-1, 2)$, $\frac{dy}{dx} = 6(-1)^2 + 8(-1) = -2$ Therefore, the equation of the tangent is $y - 2 = -2(x+1)$ $y = -2x$	Correct gradient of the tangent.	Correct equation.	

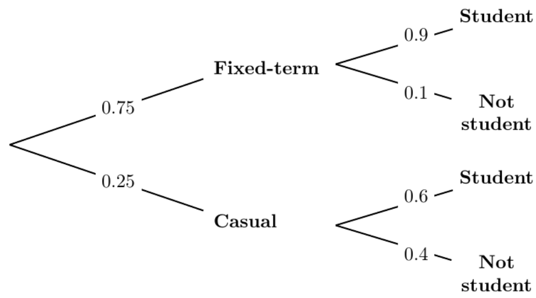
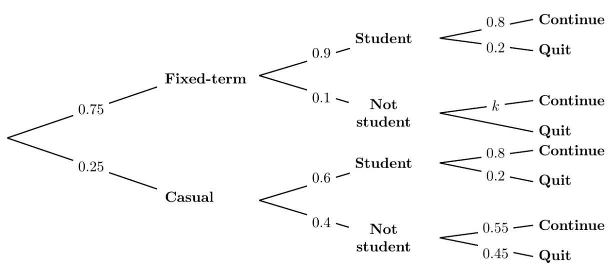
Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(c)	$6\times4x+5\times5y=100$ $y=\dfrac{100-24x}{25}$ $A=xy$ $=\dfrac{1}{25}x(100-24x)$ $=\dfrac{1}{25}(100x-24x^2)$ $\dfrac{dA}{dx}=\dfrac{4}{25}(25-12x)$ Solving $\dfrac{dA}{dx}=0$ gives $x=\dfrac{25}{12}$ (Accept $x = 2.083$.) Therefore, the maximum possible area of one of the smaller fields is $A=\dfrac{1}{25}\left(\dfrac{25}{12}\right)\left(100-24\times\dfrac{25}{12}\right)=\dfrac{25}{6}\ \text{m}^2$ (Accept $A = 4.17\ \text{m}^2$)	ONE of: • the area function in terms of x consistently derived. • correct first derivative of the area function found.	x or y value of the maximum area found.	Maximum area found with justification.

N1	N2	A3	A4	M5	M6	E7	E8
Attempt at ONE question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

N0 = No response; no relevant evidence.

Assessment Schedule – 2021 v1
Mathematics and Statistics: Apply probability methods in solving problems (91267)
Evidence Statement

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a) (i)	628 – 160 ×2 = 308 g 628 + 160 ×2 = 948 g Between 308 g and 948 g.	Correct range.		
(a) (ii)	$P(536 < X < 720)$ $= P(-0.575 < Z < 0.575)$ $= 0.4348$	Correct probability.		
(a) (iii)	$P(X < 348)$ $= P(Z < -1.75)$ $= P(Z > 1.75)$ $= 0.5 - 0.4599$ $= 0.0401$ $P(X < 348)^2 = 0.0401^2 = 0.001608$	Correct $P(X < 348)$.	Correct final probability calculated.	
(a) (iv)	The estimated probability of obtaining a large cauliflower is 9/120 = 0.075. $P(X \geq k) = 0.075$ $P\left(Z \geq \frac{k-628}{160}\right) = 0.075$ $P\left(0 \leq Z \leq \frac{k-628}{160}\right) = 0.425$ $\frac{k-628}{160} = 1.439$ $k = 1.439 \times 160 + 628$ $= 858.24$ Large cauliflowers are greater than 858.24 g.	Calculated the correct probability of a cauliflower being large as 0.075.	Correct set-up of probability statement.	Correct range found.

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a) (i)	 P(Fixed-term and not a student) $= 0.75 \times 0.1 = 0.075$	Probability correct. Probability tree is not required.		
(a) (ii)	$0.75 \times 0.9 + 0.25 \times 0.6 = 0.825$	Probability correct.		
(a) (iii)	P(Not a student) = $1 - 0.825 = 0.175$ Also accept the calculation $0.75 \times 0.1 + 0.25 \times 0.4 = 0.175$ P(Casual and not a student) = $0.25 \times 0.4 = 0.1$ P(Casual | not a student) = $0.1/0.175 = 0.5714$	Either numerator or denominator is correct. Allow consistency with their clearly drawn tree diagram.	Correct conditional probability.	
(b) (i)	Let the required probability be *k*.  $0.75 \times 0.9 \times 0.8 = 0.54$ $0.75 \times 0.1 \times k = 0.075k$ $0.25 \times 0.6 \times 0.8 = 0.12$ $0.25 \times 0.4 \times 0.55 = 0.055$ Since the sum of these four probabilities is equal to 0.7638 $0.54 + 0.075k + 0.12 + 0.055 = 0.7638$ $0.075k = 0.0488$ $k = 0.651$		Clearly drawn extended tree diagram. AND Consistently calculated probability.	Correct probability calculated.

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a) (i)	$\frac{85}{305} = 0.2787$ or equivalent.	Proportion found.		
(a) (ii)	$\frac{195+61}{305} = \frac{256}{305} = 0.8393$ Expected number is 25 × 0.8393 = 21 Accept 20 days.	Expected number found.		
(a) (iii)	$P(\text{loss} \mid \text{winter}) = \frac{26}{50} = 0.52$ $P(\text{loss} \mid \text{summer}) = \frac{1}{90} = 0.0111$ Relative risk of making a loss in winter, compared in summer is 0.52 / 0.0111 = 46.8 Mike is 46.8 times more likely to make a loss in winter than he is in summer.	One risk correctly calculated.	Correct relative risk calculated.	Correct relative risk and its explanation.
(a) (iv)	• Daily running cost may change this year, therefore last year's estimates may become unreliable if customer demand is the same as last year. • Since the number of days for each season in the table is different, making estimations for this year using the table is likely to give inflated/deflated numbers. *Accept any other valid reason.*	Clearly explained ONE reason.	Clearly explained TWO reasons.	