2021 Year 10 /10A Topic Tests Information Sheet

2021 Year 10/10A Topic Tests is a set of short answer questions and their solutions.

The topics covered are:

Year 10 topics
● Financial Mathematics (3 questions)
● Indices and Numbers of Any Magnitude (2 questions)
● Algebraic Expressions and Indices (3 questions)
● Linear Relationships (3 questions)
● Quadratics and Non-linear Relationships (3 questions)
● Measurement (2 questions)
● Trigonometry (3 questions)
● Geometrical Figures (3 questions)
● Probability 1 (2 questions)
● Probability 2 (2 questions)
● Single Variable Statistics (3 questions)
● Single Variable and Bivariate Statistics (2 questions)

Year 10A topics
● Indices and Surds (2 questions)
● Expressions, Equations and Linear Relationships (2 questions)
● Measurement (2 questions)
● Quadratic Expressions, Quadratic Equations and Parabolas (3 questions)
● Trigonometry (2 questions)
● Geometrical Figures (2 questions)
● Non-linear Relationships, Functions and Their Graphs (2 questions)
● Logarithms and Polynomials (2 questions)
● Single Variable and Bivariate Statistics (2 questions)

2021 Year 10 Topic Tests also includes a **Year 10 Mid-year test** that consists of:
● 15 multiple choice questions
● 5 short answer questions
● 2 extended response questions

Starting from 2021, we are offering a separate **Year 10A Mid-year test** so that 10A students can be assessed more thoroughly on 10A topics.

We have prepared more tests and questions this year so that it is easier for teachers to assess students topic by topic. This is the new format that we are following from now on.

2021 Year 10 Mathematics
Financial Mathematics Test

Time allowed: 1 hour
Total marks: 25 marks

Question 1 (10 marks)

a. Marcus earns $427.50 for 38 hours of work.　　　　　　　　　　　　　　　　2 marks
　What is his hourly pay rate?

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b. A holiday package is normally priced at $8,520.　　　　　　　　　　　　　　2 marks
　If this package is discounted by 12%, what will be the discounted price?

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c. Dean works as a salesman who earns an annual salary of $74,516, plus 5% commission on his sales.

　　i. What is Dean's base fortnightly salary, before the commission on his sales is added?　　2 marks

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　　ii. Dean's sales were $2,700 last week.　　　　　　　　　　　　　　　　2 marks
　　　What was his gross income last week?

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Question 2 (8 marks)

a. A $7,800 loan is borrowed to finance a car loan at 10.5% per annum simple interest for 5 years.

    i. What will be the total amount of interest after 5 years?   2 marks

_____

_____

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    ii. The repayments of the car loan are made monthly.   2 marks
       How much is each monthly repayment?

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b. Tom's loan is charged simple interest at a rate of 8.5% per annum, and has a life of 6 years.   2 marks
The total amount of interest Tom will pay on his loan is $229.50.
What is the principal of Tom's loan?

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c. Kelly wants to invest $6000 at 4% per annum simple interest.   2 marks
After how many years will the return on her investment exceed 25% for the first time?

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Question 1 (10 marks)

a. Simplify $2a^2 \times a^{-2}$.　　　　　　　　　　　　　　　　　　　　　　　　　2 marks

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b. Write $2 \times x^{-1} \times x^2 \times x \times z$ in index form.　　　　　　　　　　　　　2 marks

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c. Simplify $\left(\dfrac{3^6}{3^2} \div 3 \right)^3$.　　　　　　　　　　　　　　　　　　　　　2 marks

Write your answer in the form 3^n, where n is an integer.

_____

_____

_____

d. Simplify $\left(\dfrac{5x^{-2}}{y} \right)^{-1}$ using positive indices only.　　　　　　　　　2 marks

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_____

e. Simplify $-2e^3 f^2 \times (-3ef^3)$.　　　　　　　　　　　　　　　　　　　2 marks

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2021 Year 10 Mathematics
Algebraic Expressions and Indices Test

Time allowed: 1 hour
Total marks: 30 marks

Question 1 (12 marks)

a. Factorise $4x^2y - 4xy^2$. 2 marks

b. Simplify $-\dfrac{x^2+4x}{7} \div \dfrac{5x+20}{14}$. 2 marks

c. Write $\dfrac{z+2}{3} - \dfrac{z+1}{7}$ as a single fraction. 3 marks

Question 2 (10 marks)

a. In the diagram below, angle ABC and angle DBE are equal.   2 marks
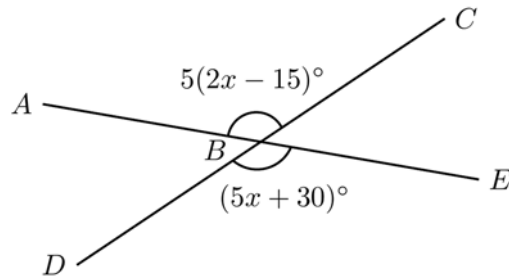
Find the value of x.

_____

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_____

b. If $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$, $u = 0.75$ and $v = 0.4$, find the value of f.   2 marks

Write your answer as a fraction in its simplest form.

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c. Solve the inequality $\dfrac{x-3}{4} + 2 \le 3$.   2 marks

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Question 1 (11 marks)
A train travels:

● 4 km in 2 minutes at a constant speed

● then at a constant speed of $\frac{8}{3}$ km per minute for 3 minutes

then stops for 1 minute to complete its journey.

a. What was the total time taken for the train to complete its journey?   1 mark
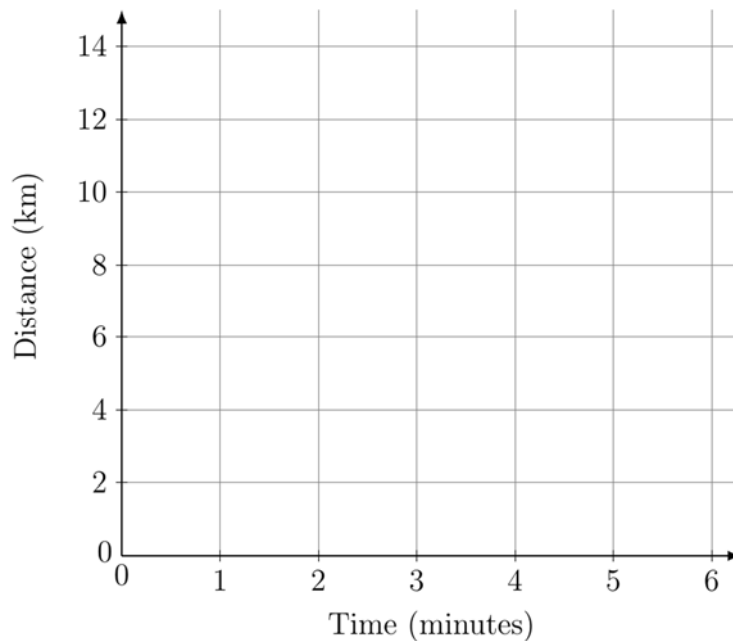
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b. What was the total distance travelled?   2 marks
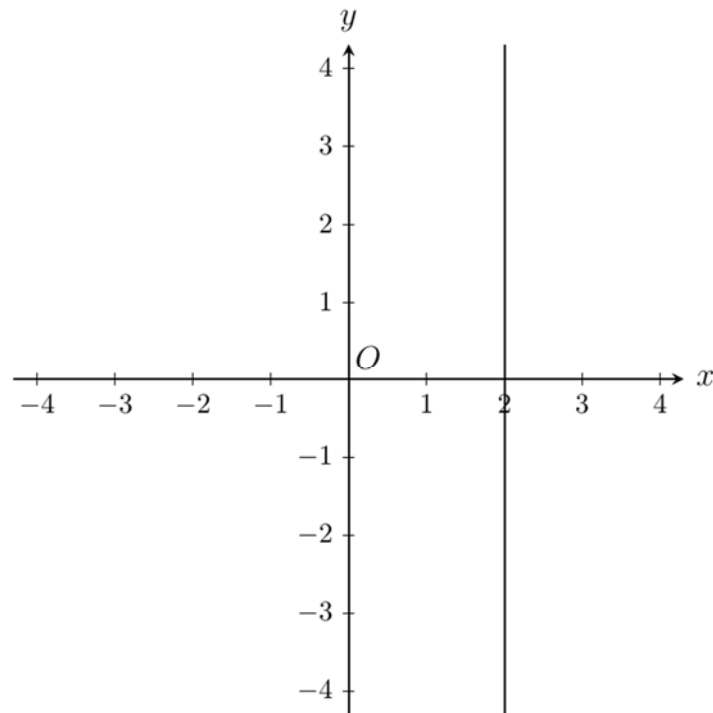
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c. Sketch a graph that represents the journey of the train on the set of axes below.   3 marks

Question 3 (5 marks)

a. Consider the straight line shown below.

    i. Write down the equation of the straight line.   1 mark

_____

_____

    ii. What is the gradient of the straight line?   1 mark

_____

2021 Year 10 Mathematics
Quadratics and Non-linear Relationships Test

Time allowed: 1 hour
Total marks: 30 marks

Question 1 (14 marks)

a. Factorise $x^2 - 20x + 64$.   2 marks

b. Factorise $5x^2 - 125$.   2 marks

c. Expand $(2y + 7)(2y - 7)$.   2 marks
Simplify your answer.

d. Simplify $\dfrac{x^2 - 6x + 9}{x - 3}$.   2 marks

Question 2 (10 marks)

a. Consider the parabola with the equation $y = (x-2)^2$.

    i. Write down the equation of the axis of symmetry.   1 mark

_____

    ii. Find the y-intercept of the parabola.   2 marks

_____

_____

    iii. Solving the equation $(x-2)^2 = 0$ for x gives $x = 2$.   1 mark
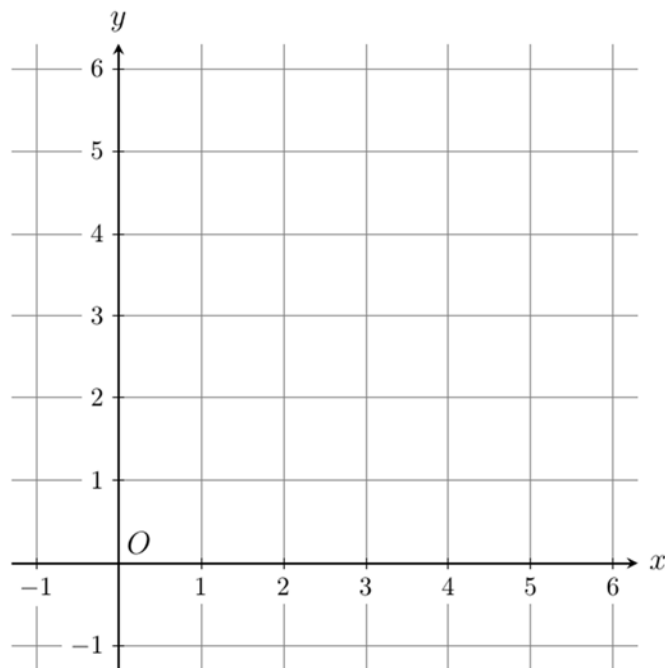   Describe what this means graphically.

_____

_____

    iv. Sketch the parabola $y = (x-2)^2$ on the set of axes below.   3 marks

    v. Find the equation of the image obtained when the graph of $y = (x-2)^2$ is reflected about   1 mark
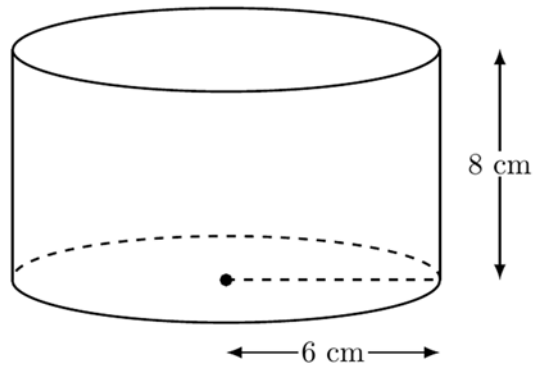   the x-axis.

_____

2021 Year 10 Mathematics
Measurement Test

Time allowed: 1 hour
Total marks: 20 marks

Question 1 (8 marks)
The following diagram shows an **open** cylindrical can.

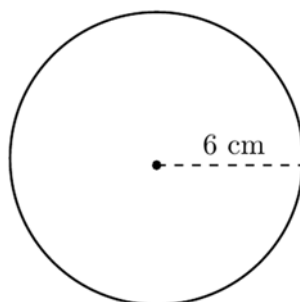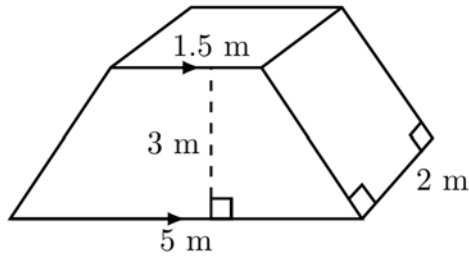**a.** Show that the circumference of the circular cross-section is 12π cm.   1 mark

_____

_____

b. Complete the diagram below for the net of the can.   2 marks
   Include any relevant measurements.

Question 2 (12 marks)

a. Consider the container shown below.
   This container is a trapezoidal prism.

i. Find the area of the cross-section of the prism.   2 marks

_____

_____

_____

ii. Find the volume of the prism.   2 marks

_____

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iii. Water is poured into the container at a rate of 600 L per minute for $\frac{7}{12}$ of an hour.   3 marks

   If the container large enough to hold all of the water?

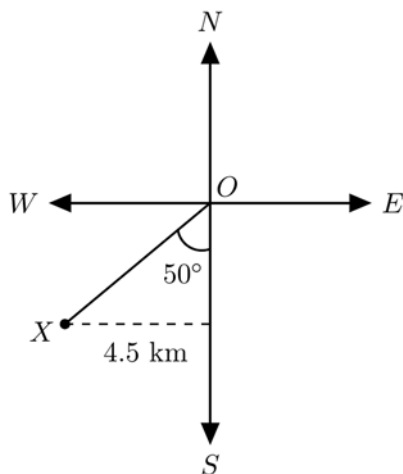   Justify your answer.

   Hint: 1 m^3 is equal to 1000 L.

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2021 Year 10 Mathematics
Trigonometry Test

Time allowed: 1 hour
Total marks: 20 marks

Question 1 (8 marks)
a. Point X is 4.5 km west from point O.

    i. Write down the bearing of O from X.   1 mark

_____

_____

    ii. Write down the bearing of X from O.   2 marks

_____

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    iii. What is the shortest distance, in **metres**, between point X and point O?   3 marks
        Round your answer to the nearest whole number.

_____

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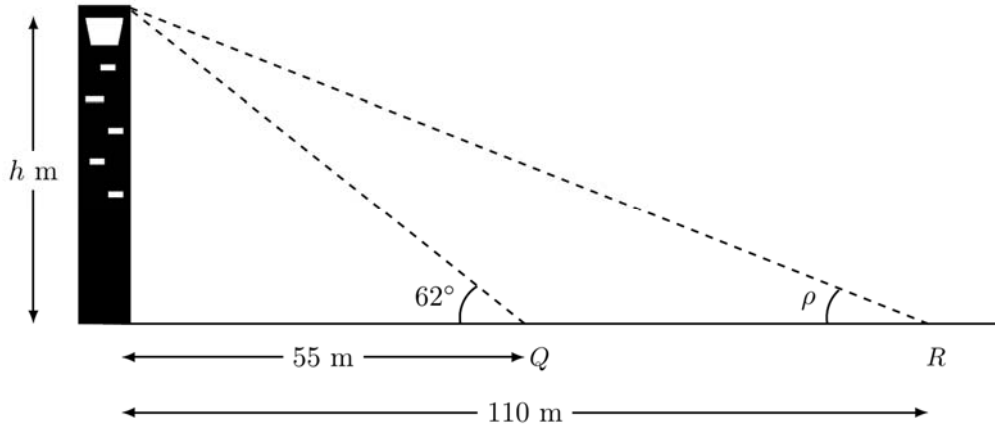
Question 3 (5 marks)

The diagram below shows a tower with height h m. Points Q and R are 55 m and 110 m away from the base of the tower respectively. The angle of elevation from point Q to the top of the tower is 62°. The angle of elevation from point R to the top of the tower is ρ.

a. Find the value of h.   2 marks

Round your answer to the nearest whole number.

_____

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_____

b. A Year 10 student claims the following: 3 marks

"Since point R is twice as far from the base of the tower as point Q, $\rho = 31°$."

Is the student's claim correct?
Justify your answer.

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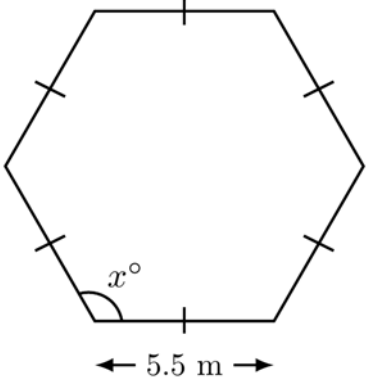_____

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Question 1 (10 marks)
a. Consider the polygon shown below.

  i. What is the name of the polygon?   1 mark

_____

  ii. Find the perimeter of the polygon.   2 marks

_____

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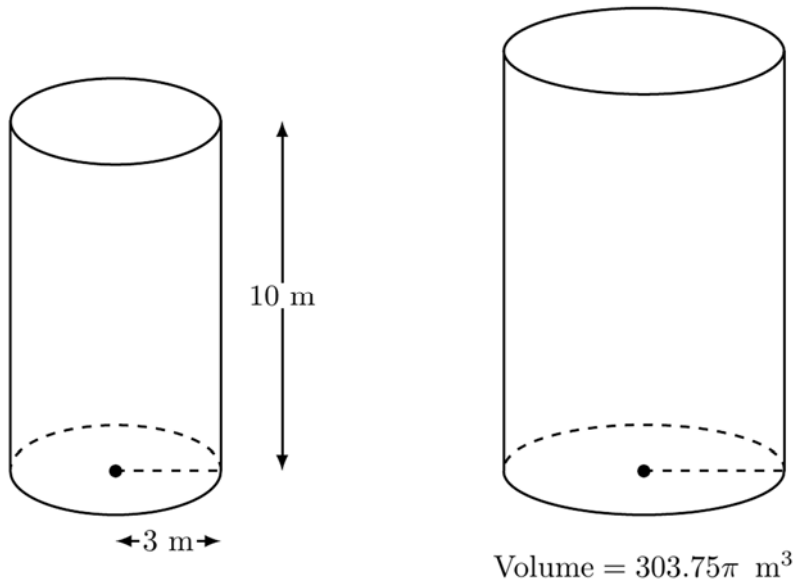
  iii. Find the value of x.   2 marks

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_____

_____

  iv. Find the size of an exterior angle of the polygon.   2 marks

_____

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_____

Question 3 (8 marks)

The following two cylinders are similar in shape.

10 m

←3 m→

$Volume = 303.75\pi \ \ m^3$

a. Find the scale factor that enlarges the smaller cylinder to the larger cylinder.   3 marks

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b. How many times greater is the surface area of the larger cylinder than the surface area of the   2 marks
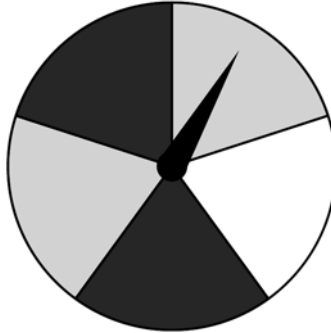smaller cylinder?

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2021 Year 10 Mathematics
Probability 1 Test

Time allowed: 1 hour
Total marks: 20 marks

Question 1 (10 marks)

The fair spinner shown below is spun 110 times, and the colour which the spinner lands on each time is recorded.

The following table shows the results.

Colour	White	Grey	Black
Frequency	17	42	51

a. How many times did the spinner land on grey or black in the experiment?   1 mark

b. What is the theoretical probability of the spinner landing on black?   1 mark

c. What is the theoretical probability of the spinner **not** landing on black?   2 marks

Question 2 (10 marks)

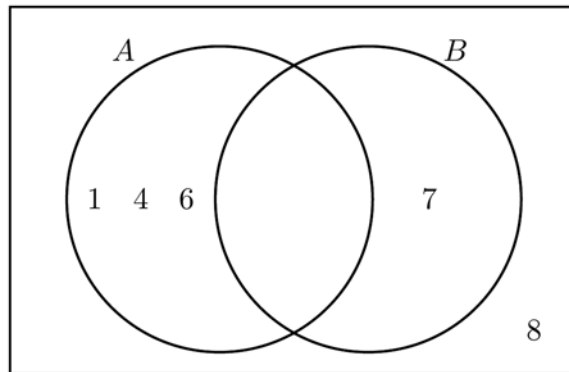A number is chosen from the set $\{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8\}$.

Let the events A and B be defined as follows.

$$A = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$$
$$B = \{2,\ 3,\ 5,\ 7\}$$

a. Complete the following Venn diagram by writing down the appropriate numbers.   1 mark

b. Describe event B in words.   1 mark

_____

_____

c. Suppose that a number is randomly selected from the first 8 positive integers.

    i. Find $P(A)$.   2 marks

      Write your answer as a fraction in its simplest form.

_____

_____

_____

    ii. Find $P(A \cup B)$.   2 marks

      Write your answer as a fraction in its simplest form.

_____

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Time allowed: 1 hour
Total marks: 20 marks

Question 1 (8 marks)

a. For events A and B, consider "the probability of A given B".

    i. Write down an expression for this probability using symbols.   1 mark

_____

    ii. State a formula used to evaluate this probability.   1 mark

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b. The two-way table below shows the number of elements in the events X and Y.

	X	not X	
Y	1	6	7
not Y	5	4	9
	6	10	16

    i. Find $P(X \cap Y)$.   1 mark

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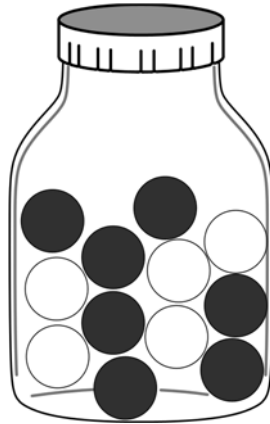   **ii.** Find $P(X \mid Y)$.   1 mark
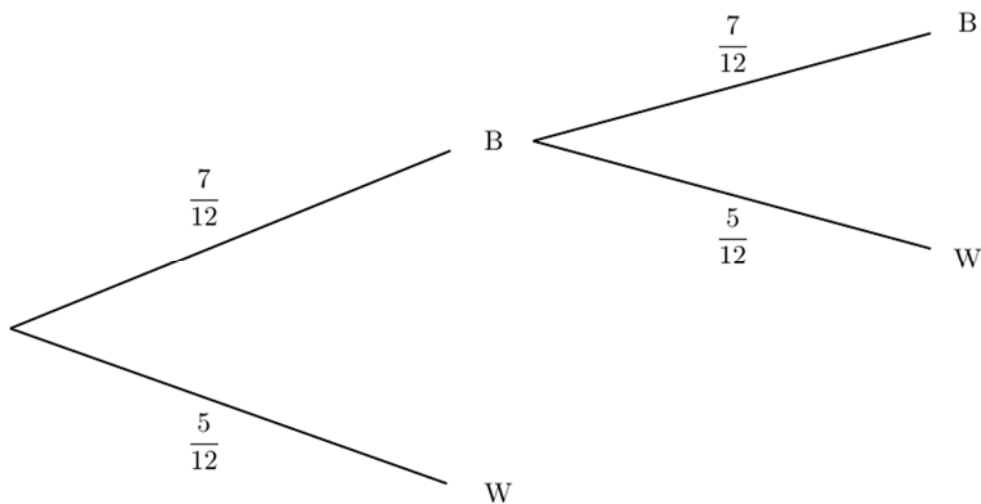
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Question 2 (12 marks)

A jar has 7 black marbles and 5 white marbles.

Two marbles are randomly drawn one after another with replacement.

a. Complete the following tree diagram by drawing in any missing features and writing down any   2 marks
missing probabilities.

b. Will the outcome of the first draw affect the outcome of the second draw?   2 marks
Justify your answer.

_____

_____

_____

2021 Year 10 Mathematics
Single Variable Statistics Test

Time allowed: 1 hour
Total marks: 25 marks

Question 1 (10 marks)

a. Here are some terms used in statistics.

| population |
| census |
| survey |
| sample |
| variable |
| confidentiality |

For each of the following descriptions, choose the most appropriate word from the list above.

i. An element or feature that can vary.   1 mark

_____

ii. All the people or objects of interest.   1 mark

_____

iii. A collection of questions designed to collect data on subsets of the population.   1 mark

_____

b. Determine the type of data each of the following questions would generate.

i. "How many cousins do you have?"   1 mark

                          Categorical   Numerical

ii. "What company is your internet service provider?"   1 mark

                          Categorical   Numerical

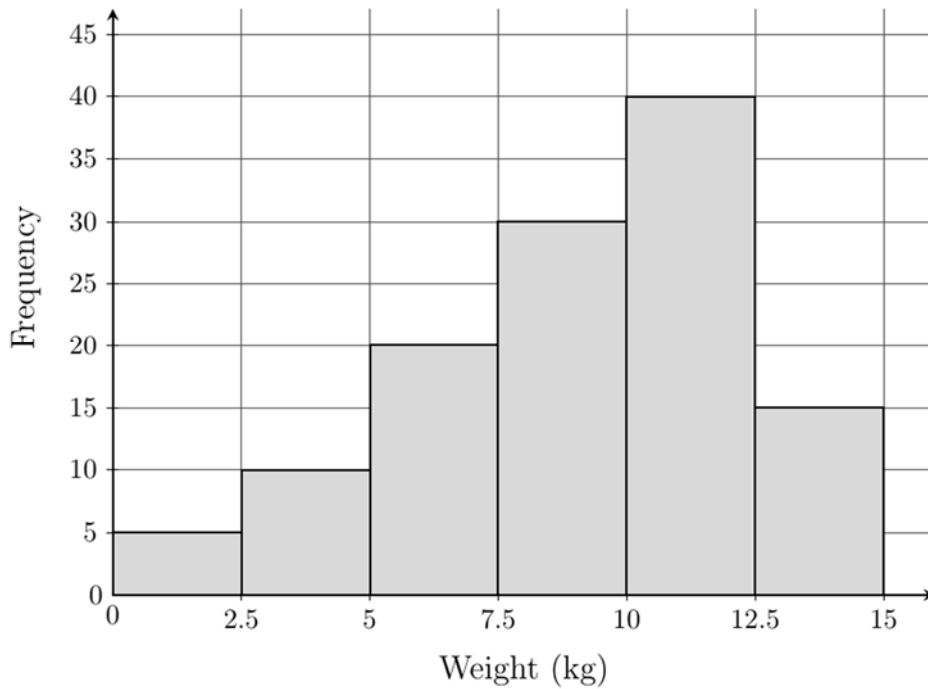**c.** An SMS survey about the number of hours high school students spend on sports is to be conducted.   2 marks
Briefly describe two potential problems with this survey.

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Question 2 (10 marks)

The following histogram shows the weight of rubbish put out by some households in Armadale last week.

a. Show that the histogram includes data from 120 households.   1 mark

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b. What percentage of households put out between 2.5 kg and 10 kg of rubbish last week?   2 marks

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_____

c. What percentage of households put out at least 5 kg of rubbish last week?   2 marks

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2021 Year 10 Mathematics
Single Variable and Bivariate Statistics Test

Time allowed: 1 hour
Total marks: 20 marks

Question 1 (10 marks)
Some students from two schools, Westlake and Avondale, were surveyed about the amount of time they usually spend per week studying Mathematics.
The results were displayed using box plots, as shown below.

Time (hours)
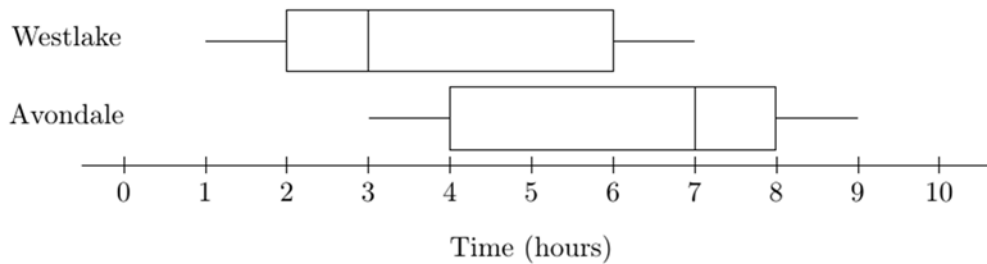
a. What percentage of Avondale students spent 4 or more hours of Mathematics per week?   1 mark

_____

b. What is the interquartile range for Westlake students?   1 mark

_____

_____

c. What statistical measure do the box plots have in common?   2 marks
  Find the value of this statistical measure.

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_____

d. At which school do students spend more time studying Mathematics per week?   2 marks
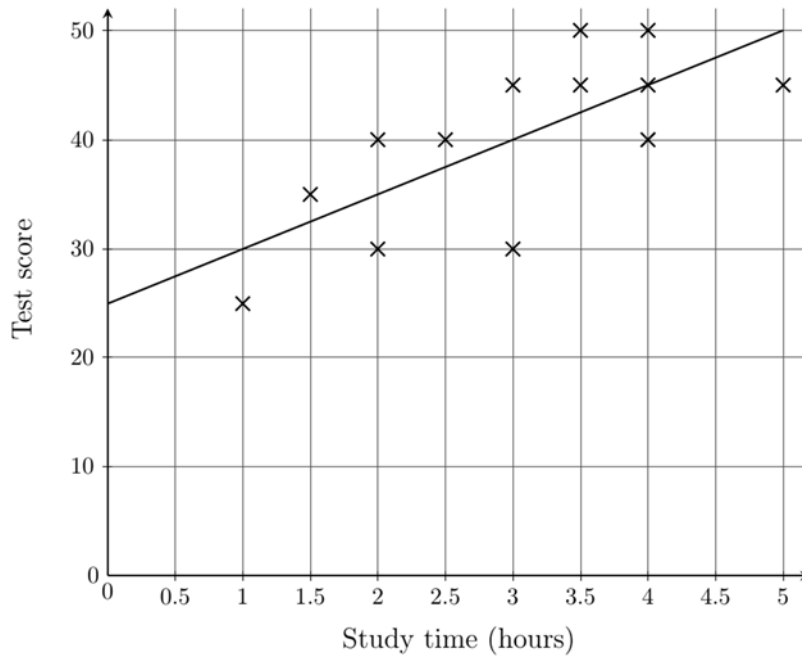 Refer to two different features of the box plots that support your answer.

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Question 2 (10 marks)

The scatter plot below shows the hours spent studying or a particular test and the score received in the test. The test was out of 50 marks.

a. State the type of the data shown by the scatter plot.   1 mark

_____

b. Briefly describe the strength of the correlation between study time and the test score.   2 marks

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c. The scatter plot shows that the study time and the test score have a positive correlation.   2 marks
Give an interpretation of this.

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2021 YEAR 10 MATHEMATICS
MIDYEAR TEST

Reading time: 15 minutes
Writing time: 2 hours

QUESTION BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of marks
A	15	15	15
B	5	5	25
C	2	2	20
			Total 60

SECTION A

Instructions for Section A

Answer **all** questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

Julia borrows $5,750 from her bank at a rate of 12% per annum compounding monthly.

The expression that gives the value of her loan at the end of 5 years is

A. $5750(1+0.12)^5$

B. $5750\left(1+\dfrac{0.12}{100}\right)^5$

C. $5750(1+0.12)^{60}$

D. $5750\left(1+\dfrac{0.12}{100}\times\dfrac{1}{12}\right)^{60}$

E. $5750\left(1+0.12\times\dfrac{1}{12}\right)^{60}$

Question 2

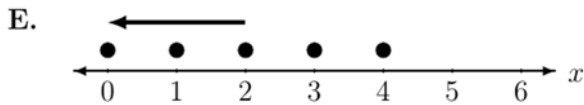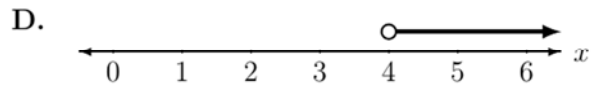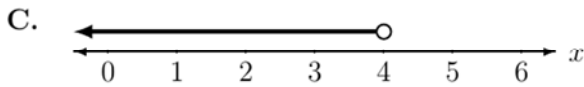Jason worked for 38 hours at a normal hourly rate of pay and 6 additional hours at double time.
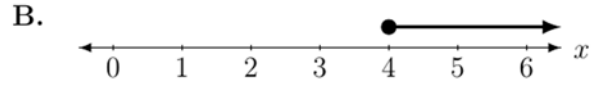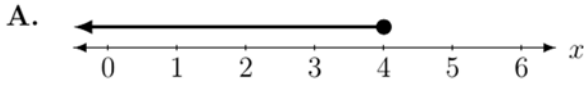
He earned $1036 in total for his work.

Jason's normal hourly rate of pay is

A. $16.71

B. $20.72

C. $23.55

D. $25.90

E. $27.50

Question 3

The number line that best represents the solution set to the inequality $x-5 \le \dfrac{2-x}{2}$, where x is a real number, is

A.

B.

C.

D.

E.

Question 4

$27x^4 - 3y^2$ is equal to

A. $9\left(3x^2 + y\right)\left(3x^2 - y\right)$

B. $3\left(3x^2 + y\right)^2$

C. $3\left(3x^2 - y\right)^2$

D. $3\left(3x^2 + y\right)\left(3x^2 - y\right)$

E. $\left(3x^2 + y\right)\left(3x^2 - y\right)$

Question 5

$\dfrac{2x+4}{3} \div \dfrac{x^2 + 4x + 4}{6}$ simplifies to

A. $\dfrac{(x+2)^3}{9}$

B. $\dfrac{4}{x+2}$

C. $\dfrac{1}{4(x+2)}$

D. $(x+2)^3$

E. $\dfrac{x+2}{4}$

SECTION A - Continued

SECTION B

┌───┐
│   **Instructions for Section B**   │
│ Answer **all** questions.   │
│ In all questions where a numerical answer is required, an **exact** value must be given unless │
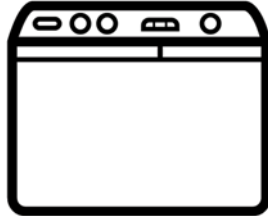│ otherwise specified.   │
│ In questions where more than one mark is available, appropriate working **must** be shown. │
│ Unless otherwise indicated, the diagrams in this book are **not** drawn to scale. │
└───┘

Question 1 (5 marks)
A washing machine worth $900 is bought in December 2018.
The value of the washing machine is depreciated by 14.5% at the end of each year, beginning in 2019.

a. Show that the value of the washing machine after n years is 900×0.855^n.   1 mark

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b. Find the value of the washing machine at the end of 2020.   2 marks

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c. In which year will the depreciated value of the washing machine first fall below $350?   2 marks
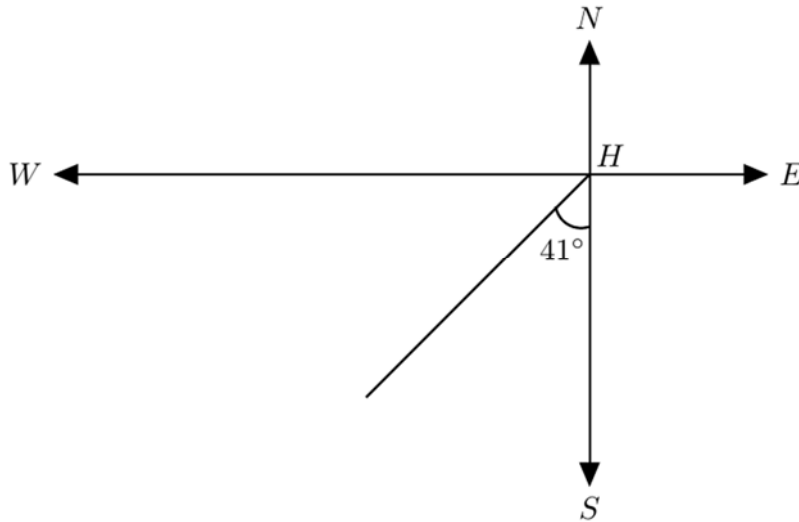
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Question 4 (5 marks)

Alice runs 1.5 km from home (*H*) on a bearing of 221º.

She then runs a further 1.8 km on a bearing of 270º to arrive at a park (*P*).

a. Complete the following diagram to show the information given above.   2 marks
   Label any relevant distances or points.

b. What is the shortest distance between Alice's home and the park?   3 marks
   Round your answer to one decimal place.

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SECTION C

| **Instructions for Section C** |
| Answer **all** questions. |
| In all questions where a numerical answer is required, an **exact** value must be given unless otherwise specified. |
| In questions where more than one mark is available, appropriate working **must** be shown. |
| Unless otherwise indicated, the diagrams in this book are **not** drawn to scale. |

Question 1 (10 marks)
The following diagram shows a photo with length 3 cm longer than its width, w cm.
The photo sits inside a frame that is 2 cm wide on each side of the photo.

a. Write down the length of the photo in terms of w.   1 mark

_____

b. Write an inequality that represents a sensible restriction on w.   1 mark

_____

c. Find an expression for the combined area of the photo and the frame in the form   2 marks

$$aw^2 + bw + c$$

where a, b and c are integers.

_____

_____

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2021 YEAR 10 MATHEMATICS MIDYEAR TEST

SOLUTIONS

SECTION A

Question	Answer
1	E
2	B
3	A
4	D
5	B
6	C
7	E
8	C
9	D
10	E
11	A
12	B
13	C
14	D
15	C

Question 1

$$5750\left(1+\frac{12}{100}\times\frac{1}{12}\right)^{12\times5}=5750\left(1+0.12\times\frac{1}{12}\right)^{60}$$

Answer is **E**.

Question 2

Let x be the normal hourly rate of pay.

$$x\times38+2x\times6=\$1036$$

$$x=\frac{\$1036}{50}$$

$$=\$20.72$$

Answer is **B**.

Question 3 (5 marks)

a. 2 marks

$2 \times \pi \times r^2 + 2 \times \pi \times r \times 2.5h$ (A1)

$= 2\pi r^2 + 5\pi rh$

$= \pi r(2r + 5h)$ (A1)

b. 3 marks

$\pi \times r^2 \times 2.5h = 2r \times 2r \times 3r$ (A1)

$2.5\pi r^2 h = 12r^3$ (A1)

$h = \dfrac{12r}{2.5\pi}$

$= \dfrac{24r}{5\pi}$ (A1)

- Accept $h = \dfrac{12r}{2.5\pi}$ as the final answer.

Question 4 (5 marks)

a. 2 marks
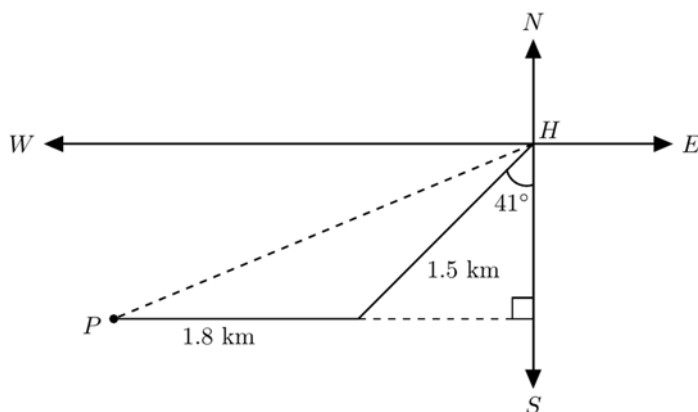
- The first part of Alice's run is labelled as 1.5 km. (A1)
- The second part of Alice's run is labelled as 1.8 km and her destination as P. (A1)

b. 3 marks

The horizontal distance between Alice's home and the park is

$\left(1.5\sin(41°) + 1.8\right)$ km (A1)

The vertical distance between Alice's home and the park is $1.5\cos(41°)$ km (A1)

Using Pythagoras' theorem

$HP = \sqrt{\left(1.5\sin(41°) + 1.8\right)^2 + \left(1.5\cos(41°)\right)^2}$

≈ 3.0 km (A1)

Time allowed: 1 hour
Total marks: 20 marks

Question 1 (10 marks)

a. Simplify $\sqrt{32} + \sqrt{18}$.   2 marks

_____

_____

_____

b. Expand and simplify $\left(2\sqrt{3} - \sqrt{2}\right)^2$.   2 marks

_____

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_____

c. The area of a circle is 72π cm².   3 marks

72π cm²
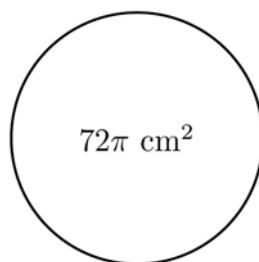
Find the radius of the circle.
Express your answer in surd form.

_____

_____

_____

_____

Question 2 (10 marks)

a. What type of number is $\left(\sqrt{2}\right)^3$?   1 mark

Circle the correct answer(s).

        Integer   Rational number   Irrational number

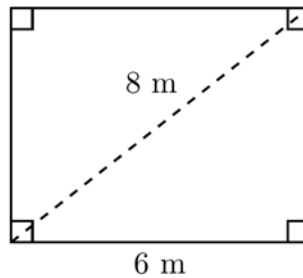**b.** Solve the equation $\dfrac{49^{2n-2}}{7^{n+1}} = 7$ for n.   3 marks

_____

_____

_____

_____

_____

_____

c.   3 marks

8 m

6 m

Find the perimeter of the rectangle above in surd form.

Express your answer in the form $a\sqrt{b}+c$, where a, b and c are integers.

_____

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Time allowed: 1 hour
Total marks: 20 marks

Question 1 (10 marks)

a. Solve the equation $\dfrac{x+1}{3} - \dfrac{x-1}{2} = 1$.   2 marks

b. Rearrange the equation $\dfrac{1}{t} = \dfrac{1}{u} - \dfrac{1}{\sqrt{v}}$ to make v the subject.   3 marks

c. Consider the equation $(a-b)x = c$, where a, b and c are constants.

Under what condition does this equation

    i. have a unique solution?   1 mark

    ii. have no solution?   1 mark

Question 2 (10 marks)

A straight line with equation $y = 2x$ is shown below.

Another line, L, that is perpendicular to this line is also shown.

The two lines intersect at the point $(2, 4)$.
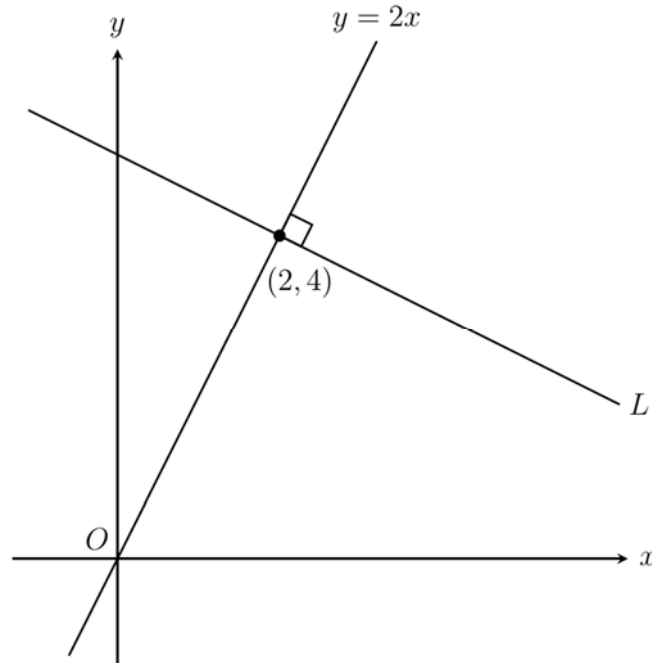
a. Find the equation of line L.   3 marks

_____

_____

_____

_____

_____

b. Find the midpoint of the line segment formed by the axis intercepts of line L.   2 marks

_____

_____

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Question 1 (8 marks)
Consider the hemisphere shown below.

$0.5R$ m
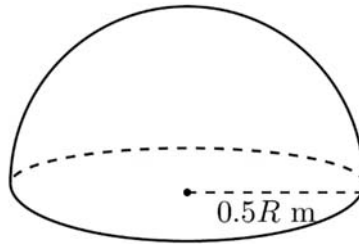
a. Find the volume of the hemisphere.   2 marks
   Write your answer in exact form in terms of R and π.

_____

_____

_____

_____

b. A sphere of radius r has the same volume as the hemisphere.   3 marks
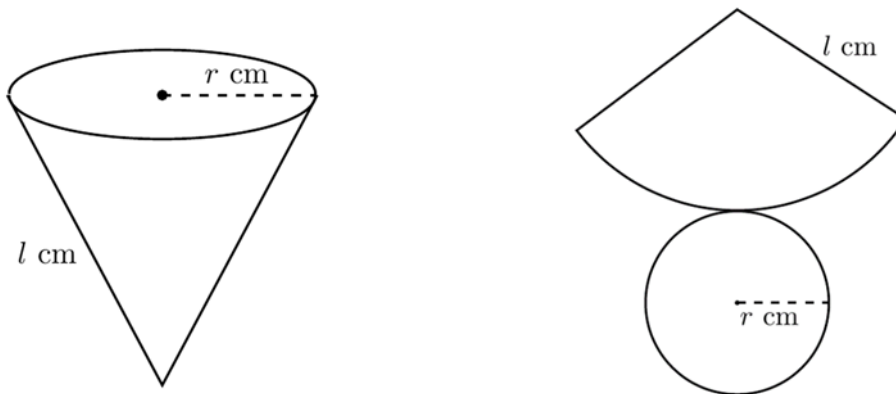   Express r in terms of R.
   Write your answer in surd form.

_____

_____

_____

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Question 2 (12 marks)

a. Consider the cone and its net diagram shown below.

    i. Show that the surface area of the cone is given by $\pi r(r+l)$ cm².   3 marks

_____

_____

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    ii. If $r=8$ and $l=17,$ find the volume of the cone.   3 marks
    Round your answer to one decimal place.

2021 Year 10A Mathematics
Quadratic Expressions, Quadratic Equations and Parabolas Test

Time allowed: 1 hour
Total marks: 30 marks

Question 1 (12 marks)

a. Expand and simplify $x(4x+2)-2x(3x-5)$. 2 marks

b. Factorise $18x^2-3x-6$. 2 marks

c. Factorise $5^2-(x+4)^2$. 2 marks

d. If $2A=2x^2-6x+1$ and $2B=4x^2-3$, find $A+B$ in terms of x. 2 marks

Question 3 (11 marks)
A rectangular field is enclosed and divided into zone A and zone B using 120 metres of string, as shown in the diagram below.
Zone A is a square with a side length of x metres, and the width of zone B is y metres.
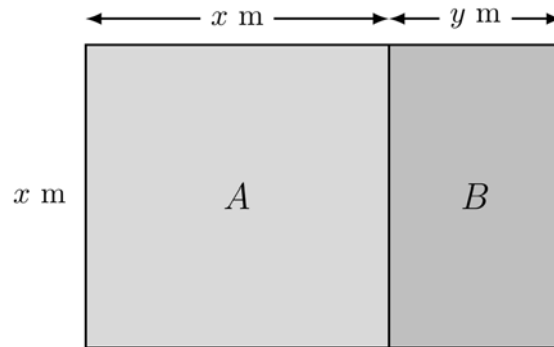
a. Find the combined area, S, of zone A and zone B in terms of x.   2 marks

_____

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b. Complete the square to show that $S = -\dfrac{3}{2}(x-20)^2 + 600$.   3 marks

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2021 Year 10A Mathematics
Trigonometry Test

Time allowed: 1 hour
Total marks: 25 marks

Question 1 (10 marks)

a. What is the exact value of $\tan(45°)$?   1 mark

_____

_____

b. For what obtuse angle θ is the equation $\sin(25°) = \sin(\theta)$ true?   1 mark

_____

_____

c. It is given that $\sin(x) = \dfrac{2}{\sqrt{5}}$ and $\cos(x) = -\dfrac{1}{\sqrt{5}}$, where $90° < x < 180°$.   2 marks

Find the exact value of $\tan(x)$.

_____

_____

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d.   **i.** Complete the graph of $y = \cos(\theta)$ for $0° \leq \theta \leq 360°$ on the set of axes below.   2 marks
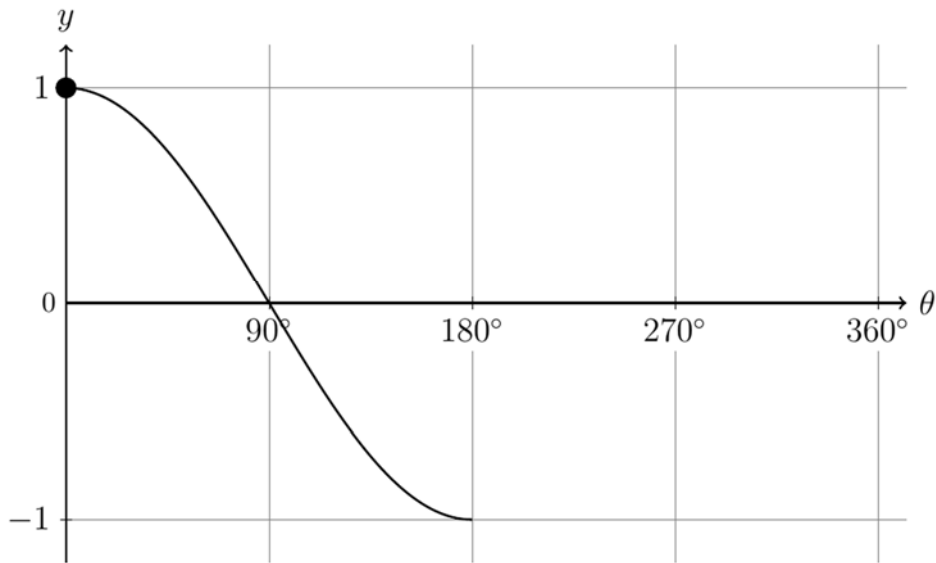
ii. Show that one solution of the equation $\cos(\theta) = 0.75$ for $0° \leq \theta \leq 360°$ is 41.4°, correct   2 marks
to one decimal place.

_____

_____

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_____

iii. Find the other solution of the equation $\cos(\theta) = 0.75$ for $0° \leq \theta \leq 360°$.   2 marks
Round your answer to one decimal place.

_____

_____

_____
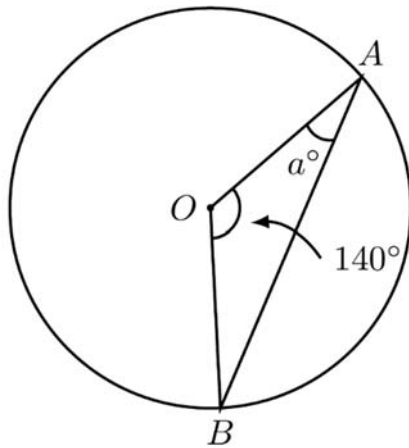
_____

In each question, ensure your responses are supported with geometric reasoning.

Question 1 (8 marks)
a. Consider the circle below.

    i. What is the size of the reflex angle at point O?   2 marks

_____

_____

_____

    ii. Find the value of a.   2 marks

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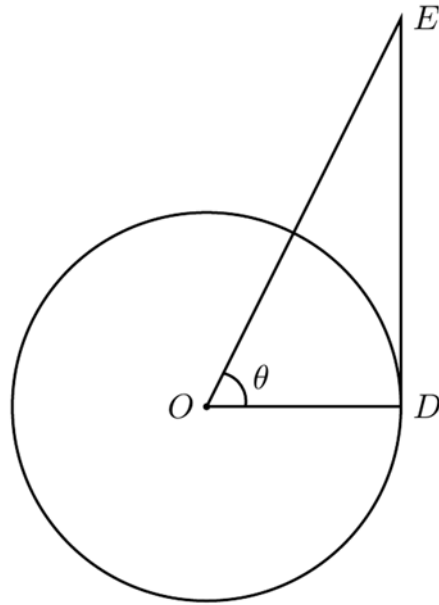**c.** In the diagram below, DE is a tangent to the circle and $\angle DOE = \theta$.   3 marks

Find $\angle OED$ in terms of θ.

_____

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Question 1 (8 marks)

a. Each of the following graphs (**i, ii** and **iii**) shows how the population, P, of a city varies over time t.
For each graph, describe whether the population is increasing or decreasing, and if the population is changing at an increasing, decreasing or constant rate.

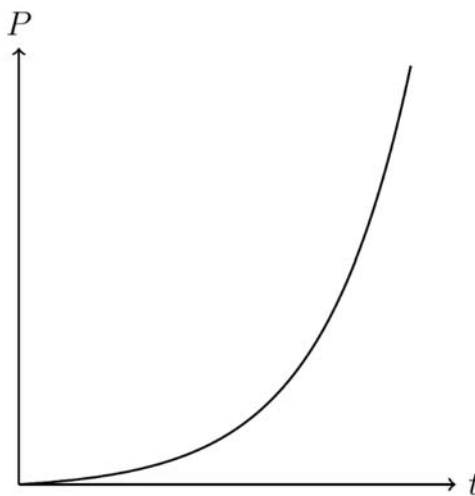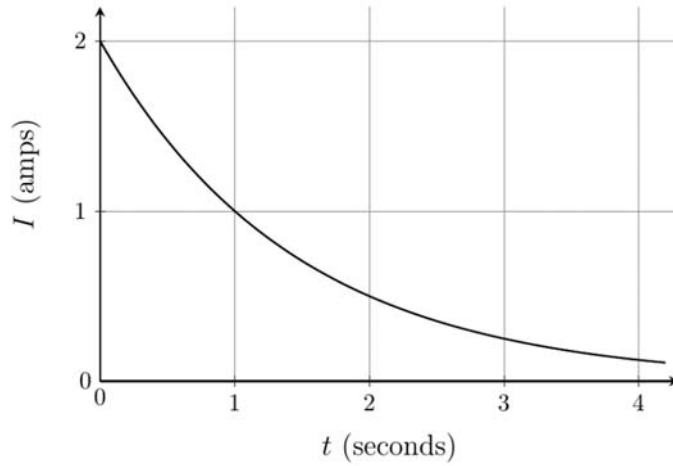   **i.**   2 marks
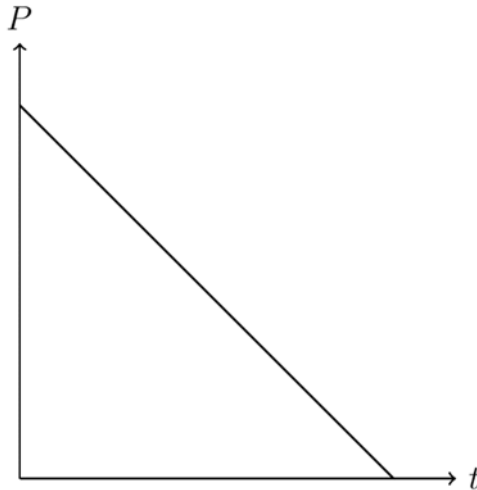
   ii.   2 marks

b. The graph below shows the amount of current flowing, I, in an electrical circuit t seconds after it is switched off.

The equation of the graph is in the form $I = a \times b^{-t}$, where a and b are integers.

i. State the values of a and b.   2 marks

_____

_____

_____

ii. How long does it take for the current to reach 0.5 amps?   2 marks

_____

_____

_____

_____

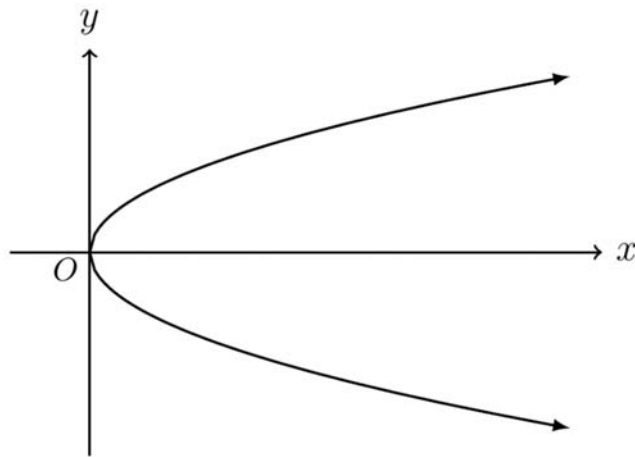**iii.** What happens to the current after a long time?   1 mark

_____

_____

d.   2 marks

Use the vertical line test to determine if the graph above represents a function.

_____

_____

_____

Question 1 (13 marks)

a. Simplify $\log_2(32)$.   2 marks

b. Simplify $\log_2\left(\dfrac{1}{2}\right) + \log_2\left(\dfrac{1}{8}\right)$.   2 marks

c. Solve the equation $4^x = 5$ using logarithms.   3 marks

d. Write $P = \log_{10}(24)$ in terms of a and b, where $a = \log_{10}(2)$ and $b = \log_{10}(3)$.   3 marks

Question 2 (12 marks)

a. Is $x^2 + \sqrt{x} + 4$ a polynomial?   1 mark
Circle the correct answer.

Yes   No

b. Consider the polynomial $Q(x) = x^3 + 2x^2 + kx - 2,$ where k is an integer.   2 marks
If $(x+1)$ is a factor of $Q(x),$ find the value of $k.$

_____

_____

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c. Consider the polynomial $P(x) = x^3 + 2x^2 + x - 2.$
Find the remainder when $P(x)$ is divided by $(x+3)$ using each of the following two methods:

i. long division.   2 marks

_____

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ii. the remainder theorem.   2 marks

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2021 Year 10A Mathematics
Single Variable and Bivariate Statistics Test

Time allowed: 1 hour
Total marks: 20 marks

Question 1 (9 marks)

a. The back-to-back stem and leaf plot below shows the maximum daily temperature for Melbourne and Sydney over a 10-day period one summer. A data summary table is also shown.

Melbourne		Sydney
	1	8 9
8 6 5 5 3	2	3 4
8 7 4 0	3	3 5 6
1	4	2 2 4

1|8 means 18°C

	Melbourne	Sydney
Minimum	23	18
Median	29	34
Mean	30.7	31.6
Maximum	41	44
Range	18	26

i. Which city would you expect to have a greater standard deviation of the maximum daily temperatures recorded?
Justify your answer. **2 marks**

_____

_____

ii. Calculate the standard deviation of the maximum daily temperatures recorded for Sydney over the 10-day period.
Round your answer to one decimal place. **2 marks**

_____

_____

_____

_____

b. In general, is the standard deviation affected by outliers? **2 marks**
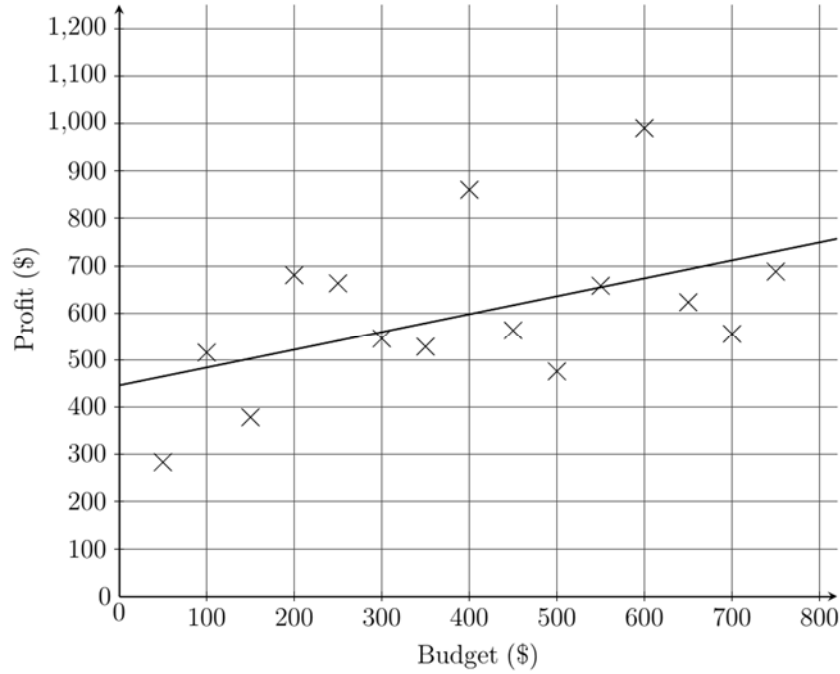Justify your answer.

_____

_____

Question 2 (11 marks)

A design business collects data on the profit made on each project and the budget allocated for marketing. This bivariate data is displayed in the scatter plot below.

A least squares line has been sketched on the scatter plot. This line passes through the point $(0, 445)$.

a. Describe the correlation between the profit made and the budget allocated for marketing in terms of strength and direction.   2 marks

_____

_____

b. Show that equation of the least squares line is   2 marks

$$\text{profit} = 445 + 0.3875 \times \text{budget}$$

_____

_____

_____

_____

_____

2021 YEAR 10A MATHEMATICS
MIDYEAR TEST

Reading time: 15 minutes
Writing time: 2 hours

QUESTION BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of marks
A	15	15	15
B	5	5	25
C	2	2	20
			Total 60

SECTION A

Question 1

Which one of the following statements about $\sqrt{4} + \sqrt{8}$ is **true**?
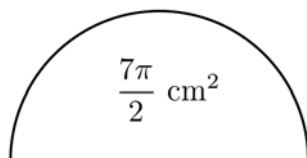
A. $\sqrt{4} + \sqrt{8}$ is an irrational number

B. $\sqrt{4} + \sqrt{8}$ is a rational number

C. $\sqrt{4} + \sqrt{8}$ is a natural number

D. $\sqrt{4} + \sqrt{8}$ is an integer

E. $\sqrt{4} + \sqrt{8}$ can be written in the form $\dfrac{a}{b}$, where a and b are integers

Question 2

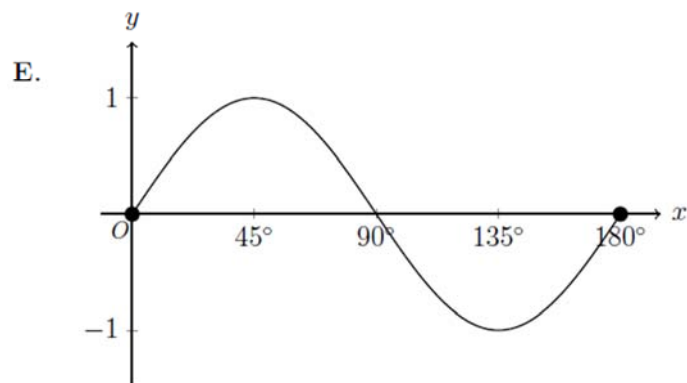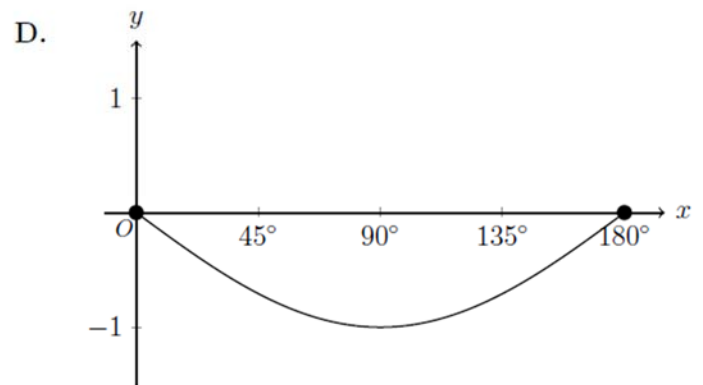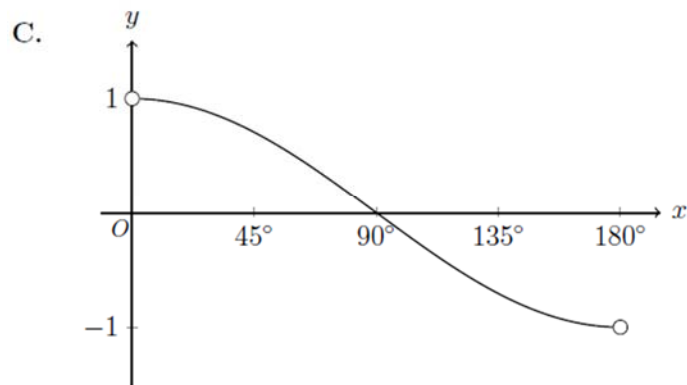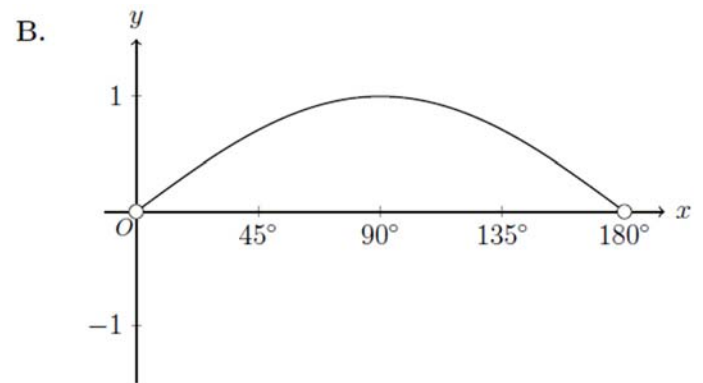If the area of a semicircle is $\dfrac{7\pi}{2}$ cm² then its radius is
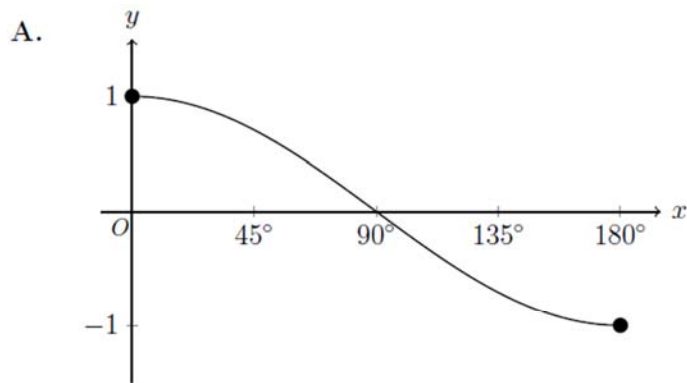
A. 1.75 cm

B. 3.5 cm

C. $\sqrt{7}$ cm

D. 7 cm

E. 14 cm

Question 10

Which graph best represents the graph of $y = \sin(x)$ for $0° < x < 180°$?

A.

B.

C.

D.

E.

SECTION B

Question 1 (5 marks)

A sheet of paper has a length of $\left(\sqrt{3}+\sqrt{12}\right)$ cm and a width of $\sqrt{7}$ cm.

A smaller rectangle with a length of $\sqrt{7}$ cm and a width of $\sqrt{3}$ cm is cut out from the middle of the paper.

$(\sqrt{3} + \sqrt{12})$ cm

$\sqrt{3}$ cm $\sqrt{7}$ cm

$\sqrt{7}$ cm
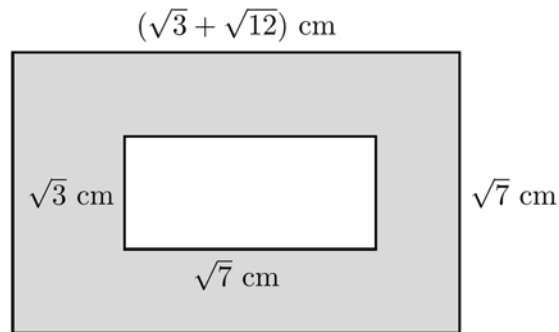
a. Find the area of the remaining paper in exact form.   3 marks

_____

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_____

b. Express the area of the remaining paper as a percentage of the original area of the paper.   2 marks
Round your answer to one decimal place.

_____

_____

_____

Question 4 (5 marks)

In the diagram below, *AB* and *CB* are tangents to the circle.

a. Find the angle θ.   3 marks

Give a reason for each step of working.

_____

_____

_____

_____

_____

b. If $AB = 4.5$ cm, what is the area of quadrilateral *OABC*?   2 marks

Round your answer to one decimal place.

_____

_____

_____

_____

_____

SECTION B - Continued

SECTION C

Instructions for Section C

Answer **all** questions.

In all questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
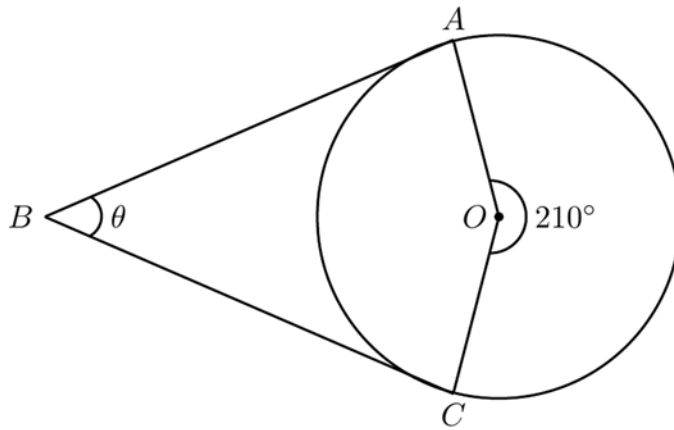
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (10 marks)

A bank charges interest on loans at the rate of 20% per annum, compounding annually.

Suppose that Luke takes up a loan of $300 with the bank at the start of 2020.

No repayments are made by Luke.

a. Find the rule for the value of Luke's loan n years after the loan was first issued in the form   1 mark
$A = a \times b^n$, where a and b are positive constants.

_____

_____

b. Calculate the value of Luke's loan at the start of 2022.   1 mark

_____

_____

_____

c. Use logarithms to find when the value of Luke's loan will exceed triple its initial value for the   3 marks
first time.

_____

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2021 YEAR 10A MATHEMATICS MIDYEAR TEST

SOLUTIONS

SECTION A

Question	Answer
1	A
2	C
3	B
4	E
5	D
6	B
7	C
8	C
9	A
10	B
11	C
12	E
13	D
14	C
15	A

Question 1

$\sqrt{4} + \sqrt{8} = 2 + 2\sqrt{2},$ which is an irrational number.

Answer is **A**.

Question 2

If r is the radius of the semicircle

$$\frac{\pi \times r^2}{2} = \frac{7\pi}{2}$$
$$r^2 = 7$$
$$r = \sqrt{7}$$

Answer is **C**.

Question 3 (5 marks)
$$ax^2 + x + c = 0$$

$$x^2 + \frac{x}{a} + \frac{c}{a} = 0 \quad \text{(A1)}$$

$$x^2 + \frac{x}{a} + \frac{1}{4a^2} - \frac{1}{4a^2} + \frac{c}{a} = 0 \quad \text{(A1)}$$

$$\left(x + \frac{1}{2a}\right)^2 = \frac{1-4ac}{4a^2} \quad \text{(A1)}$$

$$x + \frac{1}{2a} = \frac{\pm\sqrt{1-4ac}}{2a} \quad \text{(A1)}$$

$$x = \frac{-1\pm\sqrt{1-4ac}}{2a} \quad \text{(A1)}$$

Question 4 (5 marks)
a.   3 marks

The obtuse angle O is $360° - 210° = 150°$ (A1)

Since AB and CB are tangents to the circle, $\angle OAB = \angle OCB = 90°$. (A1)

Since the internal angles of a quadrilateral sum to 360º

$$\theta + 90° + 90° + 150° = 360°$$

$$\theta = 30° \quad \text{(A1)}$$

b.   2 marks

Triangle OCB and triangle OAB are identical right-angled triangles. (A1)

Therefore, the area of quadrilateral $OABC$ is

$$2 \times \frac{1}{2} \times 4.5 \times 4.5 \tan(15°)$$

$$\approx 5.4 \text{ cm}^2 \quad \text{(A1)}$$

Question 2 (10 marks)
a.   2 marks
The surface area of the composite solid is
$(2 \times 3y \times 3y) + (4 \times 3y \times y) + 2\pi \times 2y \times 2y$ (A1)
$= 30y^2 + 8\pi y^2$
$= 2(15 + 4\pi)y^2 \text{ cm}^2$ (A1)

b.   2 marks
The volume of the composite solid is
$\left(\pi(2y)^2 \times 2y\right) + (3y \times 3y \times y)$ (A1)
$= 8\pi y^3 + 9y^3$
$= (8\pi + 9)y^3 \text{ cm}^3$ (A1)

c.   3 marks
$\frac{4}{3}\pi(ky)^3 = (8\pi + 9)y^3$ (A1)

$\frac{4}{3}\pi k^3 y^3 = (8\pi + 9)y^3$

$\frac{4}{3}\pi k^3 = (8\pi + 9)$

$k^3 = (8\pi + 9) \times \frac{3}{4\pi}$ (A1)

$k = \sqrt[3]{(8\pi + 9) \times \frac{3}{4\pi}}$

≈ 2 (A1)

d.   3 marks
The initial (full) volume of the container is
$(8\pi + 9) \times (4.1)^3 \text{ cm}^3$ (A1)

The volume remaining in the container after t minutes is
$\left((8\pi + 9) \times (4.1)^3 - dt\right)$ (A1)
where d is the rate at which the water is being drained.

Therefore
$(8\pi + 9) \times (4.1)^3 - d \times 10 = 1879$

$$d = \frac{1879 - (8\pi + 9) \times (4.1)^3}{-10}$$

$\approx 47 \text{ mL per minute}$ (A1)