2

9 1 2 6 1

Level 2 Mathematics and Statistics, 2020 v1

91261 Apply algebraic methods in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. Guess-and-check methods, and correct answer(s) only, will generally limit grades to Achievement.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Solve each of the following equations for x:

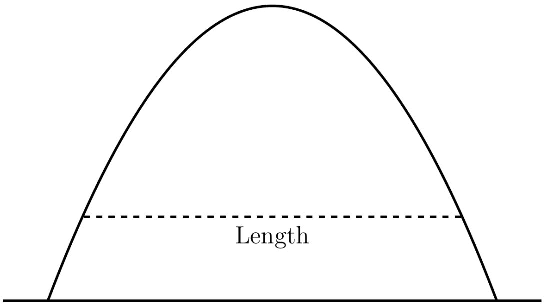
(i) $8x^2 = 14x + 15$

(ii) $\left(1 - \frac{2}{x}\right)\left(1 + \frac{2}{x}\right) = \frac{7}{x}$

(b) Factorise fully $2ab - 2cd - ac + 4bd$.

(d) Gary puts together a tent on a camping trip on a flat surface.

https://www.letgo.com/en-tr/i/6-man-tent-rei- base-camp -6_fe37451b-7e51-480c-a557-cbe99f43b5cd



The height h in centimetres of the tent can be modelled by the function $y = a(x-120)^2 + 144$, where x is the position of the tent in centimetres from its bottom left corner.

What is the length of the tent 100 centimetres above the surface?

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Write the expression $\frac{2}{c+6} - \frac{c+1}{c^2+5c-6}$ as a single fraction in its simplest form.

(b) Find where the graph $y = \log_3(x-3) - 2$ cuts the x-axis.

ASSESSOR'S USE ONLY

(c) The size of the population P of a fish in a lake continuously increases exponentially.

The model $P = A \times b^t$ describes the fish population at the end of t years.

The size of the fish population at the start of 2011 was 2500.

(i) It has been claimed that the population of the fish grows at a rate of 15% per year.
How long will it take for the fish population to reach 5000 at this rate?

(ii) It has been suggested by experts that a more accurate exponential growth rate is r% per year.
The fish population at the end of 2015 is 3790.
Find the value of r.

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(d)	$\dfrac{x^2+2px+1}{qx^2-4x-1}=k$ $x^2+2px+1=k(qx^2-4x-1)$ $(1-kq)x^2+(2p+4k)x+1+k=0$ The quadratic equation in the last line must be an identity for x. This means $k=-1$, $p=2$ and $q=-1$.	Equated the expression to a constant.	Correctly simplified the equation in u to a quadratic equation. AND Either p or q value is correct.	Correct values for both p and q.
(e)	$\Delta=\left(-2(a+k)\right)^2-4\times1\times(10-a)$ $=4(a+k)^2-4(10-a)$ $=4\left((a+k)^2+a-10\right)$ The equation has real roots if $\Delta\geq0$. Since $(a+k)^2\geq0$ is always true, we require $a-10\geq0$ or $a\geq10$. Therefore, the minimum value of a is 10.	Correct discriminant derivation.	Recognised that the term $(a+k)^2$ is always equal to or greater than 0.	Correct answer.

N1	N2	A3	A4	M5	M6	E7	E8
Attempt at ONE question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

N0 = No response; no relevant evidence.

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\frac{y}{x^2}\times\left(xy^2\right)^{-3}=\frac{y}{x^2}\times\frac{1}{x^3y^6}$ $=\frac{1}{x^5y^5}$ $=(xy)^{-5}$ Therefore $n=5$.	Correct solution.		
(b)	$\log_3\left(27^{-1}\right)+\log_3(9^{-2})$. $=\log_3\left(3^{-3}\right)+\log_3(3^{-4})$ $=-3-4$ $=-7$	Correct answer.		
(c)	$\log_2\left(2x+1\right)-\log_2\left(x-1\right)=2$ $\log_2\left(\frac{2x+1}{x-1}\right)=2$ $\frac{2x+1}{x-1}=2^2$ $2x+1=4x-4$ $-2x=-5$ $x=\frac{5}{2}$	Derived a non-log equation (third line).	Correct solution.	

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\frac{2}{c+6}-\frac{c+1}{c^2+5c-6}=\frac{2(c-1)}{(c+6)(c-1)}-\frac{c+1}{(c+6)(c-1)}$ $=\frac{2(c-1)-(c+1)}{(c+6)(c-1)}$ $=\frac{2c-2-c-1}{(c+6)(c-1)}$ $=\frac{c-3}{(c+6)(c-1)}$	Correct terms with the common denominator.	Correct final expression.	
(b)	Set $y=0$ and then solve for x. $\log_3(x-3)-2=0$ $\log_3(x-3)=2$ $x-3=3^2$ $x=12$	Correct solution.		
(c) (i)	$P=2500\times1.15^t$ $2500\times1.15^t=5000$ $1.15^t=2$ $t\log(1.15)=\log(2)$ $t=\frac{\log(2)}{\log(1.15)}$ ≈4.96 After 4.96 years.	Wrote down the correct equation.	Correct answer supported with correct steps.	
(c) (ii)	$2500\times\left(1+\frac{r}{100}\right)^5=3790$ $\left(1+\frac{r}{100}\right)^5=1.516$ $1+\frac{r}{100}=\sqrt[5]{1.516}$ $r=100\times\left(\sqrt[5]{1.516}-1\right)$ ≈8.678	Wrote down the correct equation.	Finds $1+\frac{r}{100}$.	Correct value of r.

<table>
<tr><th>Q</th><th>Expected Coverage</th><th>Achievement (u)</th><th>Merit (r)</th><th>Excellence (t)</th></tr>
<tr><td>(d)</td><td>Since a and b are the roots
$a^2 - 2ma + 1 = 0$ or $a^2 + 1 = 2ma$ (1)
$b^2 - 2mb + 1 = 0$ or $b^2 + 1 = 2mb$ (2)

Also, the quadratic equation can be written as
$(x-a)(x-b) = 0$
$x^2 - (a+b)x + ab = 0$

Comparing the coefficients of this quadratic equation with those given in the question
$a + b = 2m$ (3) and $ab = 1$ (4)

Therefore
$\frac{(a^2+1)(b^2+1)}{a^2+b^2+4}$
$= \frac{(a^2+1)(b^2+1)}{(a+b)^2 - 2ab + 4}$
$= \frac{2ma \times 2mb}{(2m)^2 - 2(1) + 4}$
$= \frac{4m^2(ab)}{4m^2+2}$
$= \frac{2m^2}{2m^2+1}$</td><td>ONE of:
• wrote down two of the four equations (1) –(4).

• simplified the given expression consistently.</td><td>Substituted appropriate equations into the given expression to make it in terms of m.</td><td>Correct final expression.</td></tr>
</table>

N1	N2	A3	A4	M5	M6	E7	E8
Attempt at ONE question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

N0 = No response; no relevant evidence.

Cut Scores

	Not Achieved	Achievement	Achievement with merit	Achievement with Excellence
Score range	0 – 7	8 – 14	15 – 20	21 – 24

2

9 1 2 6 2

Level 2 Mathematics and Statistics, 2020 v1

91262 Apply calculus methods in solving problems

Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You must show the use of calculus in answering all questions in this paper.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

QUESTION ONE

(a) A function f is given by $f(x) = 3x^3 - 2x + 4$.
Find the gradient of the graph of the function at the point where $x = 2$.

(b) An object starts from a fixed point and moves along a straight line.
At time t seconds, the object's displacement s metres from the fixed point is modelled by the function

$$s(t) = 10t^2 - t^3$$

(i) What is the object's velocity at $t = 2$?

(ii) Use calculus to find the maximum velocity of the object.

QUESTION TWO

ASSESSOR'S USE ONLY

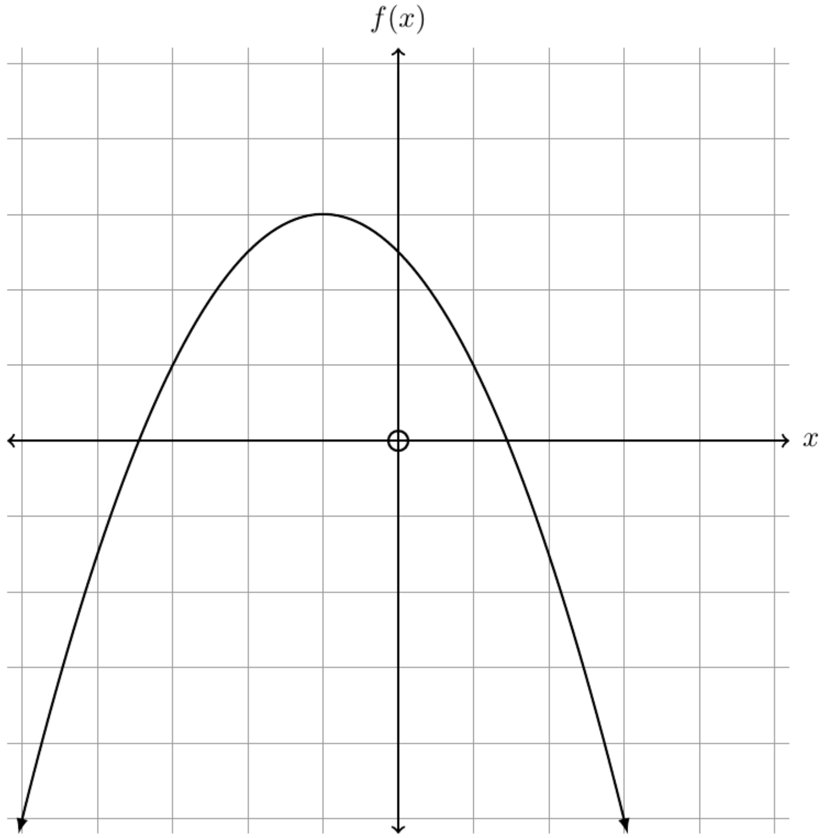
(a) The gradient function of a curve is $f'(x) = -3(x^2 + 1)$.

The point $(-1, 5)$ is on the curve.

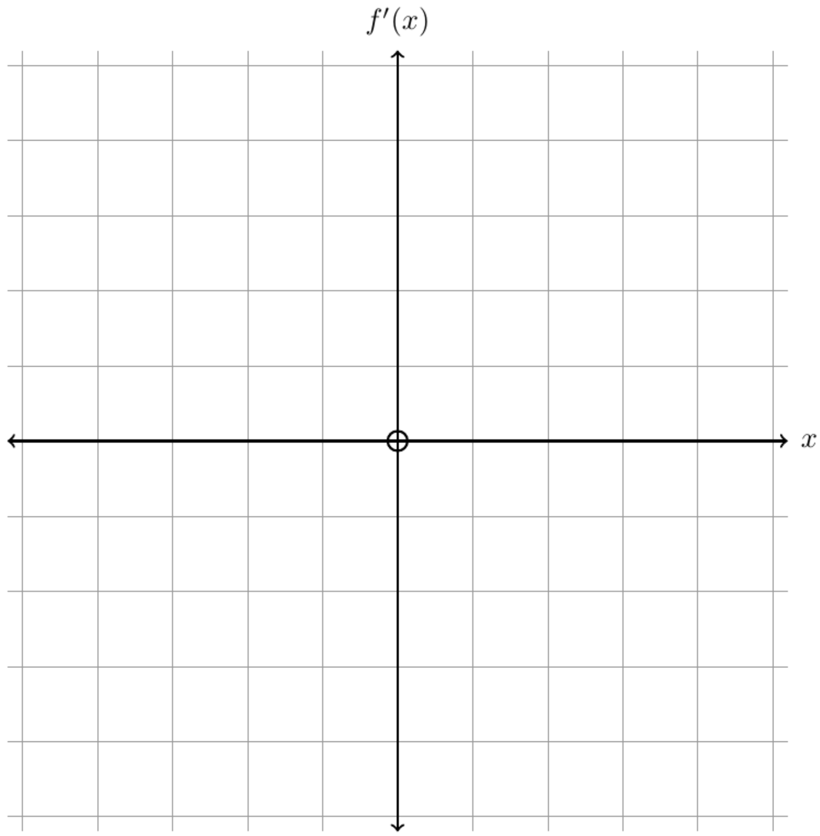
Find the equation of $f(x)$.

(b) Find the x-coordinate(s) of the point(s) on the graph of the function $y = 4x^3 + 18x^2 + 27x + 50$ where the tangent to the curve is parallel to the line $y - 12x - 4 = 0$.

(d) The diagram below shows the graph of the function $y = f(x)$.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.



ASSESSOR'S USE ONLY

(e) Consider the graph of the function $y = a(x-4)(x+6)$, where a is a non-zero constant.

Use calculus, where possible, to investigate each of the following:

- the coordinates of the turning point in terms of a
- the local maximum/minimum value
- the range of x values for which the function is increasing.

ASSESSOR'S USE ONLY

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$f(x) = -x^3 - 3x + C$ Substituting $(-1, 5)$ $f(-1) = 5$ $-(-1)^3 - 3(-1) + C = 5$ $1 + 3 + C = 5$ $C = 1$ $f(x) = -x^3 - 3x + 1$	Correct $f(x)$.		
(b)	$\frac{dy}{dx} = 12x^2 + 36x + 27$ $= 3(4x^2 + 12x + 9)$ $= 3(2x+3)^2$ The gradient of the line $y - 12x - 4 = 0$ is 12. $3(2x+3)^2 = 12$ $(2x+3)^2 = 4$ $2x + 3 = \pm 2$ $2x = -3 \pm 2$ $x = -\frac{5}{2}, -\frac{1}{2}$	Correct derivative $\frac{dy}{dx}$.	Correct x values found.	
(c)	$A = 4\pi r^2$ $= 4\pi \left(\frac{t}{4}\right)^2$ $= \frac{\pi t^2}{4}$ $\frac{dA}{dt} = \frac{\pi t}{2}$ $t = 10$ when $r = 2.5$, therefore $\frac{dA}{dt} = \frac{\pi \times 10}{2}$ $= 5\pi \text{ cm}^2/\text{s}$ Accept 15.71 cm^2/s.		Relationship between A and t formed and the derivative $\frac{dA}{dt}$ found.	Rate of change of surface area calculated. Units not required.

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\frac{dP}{dn} = -12n + 2400$ Max profit occurs when $\frac{dP}{dn} = 0$. Therefore $-12n + 2400 = 0$ $-12n = -2400$ $n = 200$ Therefore, the maximum profit is $P = -6(200)^2 + 2400(200) - 6000$ $= \$234{,}000$	Derivative $\frac{dP}{dn}$ found and equated to 0.	The value of n for maximum profit and the maximum profit are found.	
(b)	Since $y = -2 \times 1 - 1 = -3$, the point $(1, -3)$ is on the curve of the graph $f(x)$. $f(x) = 2x^2 - 6x + C$, where C is a constant. $2(1)^2 - 6(1) + C = -3$ $2 - 6 + C = -3$ $C = 1$ Therefore, $f(x) = 2x^2 - 6x + 1$.	Correct $f(x)$.		
(c)	$f'(x) = 3x^2 - 6x - 9$ $= 3(x^2 - 2x - 3)$ $= 3(x-3)(x+1)$ Solving $f'(x) = 0$ gives $x = -1$ or $x = 3$. $f'(-2) > 0$, $f'(0) < 0$ and $f'(4) > 0$. Therefore, the minimum value of the function occurs at $x = 3$. The minimum value is $f(3) = 3^3 - 3(3)^2 - 9(3) + 1 = -26$.	Correct derivative and turning points found.	Used either the first derivative or the second derivative test to determine the nature of both turning points. AND Justified which turning point is the minimum and found the correct minimum value.	

2

9 1 2 6 7

Level 2 Mathematics and Statistics, 2020 v1

91267 Apply probability methods in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

QUESTION ONE

ASSESSOR'S USE ONLY

The doctors who work at the emergency unit of a hospital are interested in obtaining some insights about their patients.
A random sample of 100 patients that were admitted to the emergency unit one weekend is selected.
Table 1 and **Table 2** show the results of the analysis the doctors performed.

Table 1

Cause of injury	Female	Male
Car accident	15	31
Fracture	13	12
Others	16	13

Table 2

Statistics	Female	Male
Number of patients	44	56
Mean age	25.7	28.1
Median age	25	27

(a) Use the information in **Table 1** to find the probability that a randomly chosen patient from this sample was:

(i) admitted to the emergency unit from a fracture.

(ii) admitted to the emergency unit from a car accident and is female.

(b) It has been claimed that males are two times more likely to be admitted to the emergency unit for a car accident than females.
Using the information in **Table 1**, explain whether the claim is correct or not.
Support your answer with appropriate calculations.

(c) The doctors at the emergency unit want to use the same data and **Table 1** to predict the proportion of patients that will be admitted to the emergency unit for fractures during one weekend in the next month.
Give TWO statistical reasons why the data may not give a valid proportion.

ASSESSOR'S USE ONLY

(b) The lightest 5% of the bottles of milk are considered underweight.
What is the maximum volume of an underweight bottle of milk?

(c) Cartons of apple juice are also advertised as containing 1 litre.

https://foodme.co.nz/p/fresh-up-apple-nectarine-juice-1l-904015122

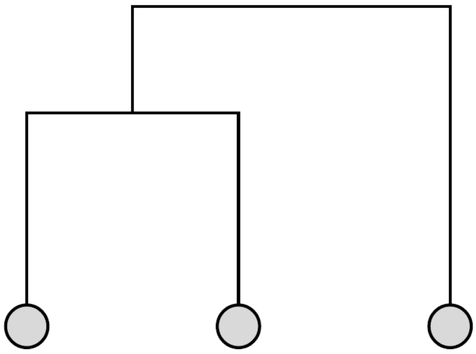
The volume of apple juice is also normally distributed with a mean of 1010 mL.
However, the volume of apple juice is known to be more variable than the volume of milk.

(i) State ONE similarity and ONE difference between the normal distribution curve of milk and the normal distribution curve of apple juice.

QUESTION THREE

Three tennis players, Demi, Jess and Wendy, are participating in a tournament.
In this tournament, two of the players are randomly paired, and they have a match.
For each match, the player could either win or lose, and the winner advances and the loser departs.

Each player will be randomly assigned into one of the three places shown in the diagram below.



The probability of Demi beating Jess is $\frac{1}{3}$, the probability of Jess beating Wendy is $\frac{1}{4}$ and the probability of Wendy beating Demi is $\frac{1}{2}$.

(a) What is the probability of Demi beating Wendy?

(b) Explain why there are three unique tournaments.

Assessment Schedule – 2020 v1
Mathematics and Statistics: Apply probability methods in solving problems (91267)
Evidence Statement

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a) (i)	$\frac{25}{100}=\frac{1}{4}=0.25$ or equivalent.	Correct probability.		
(a) (ii)	$\frac{15}{100}=\frac{3}{20}=0.15$ or equivalent.	Correct probability.		
(b)	$\text{P}(\text{car accident} \mid \text{males})=\frac{31}{56}=0.5536$ $\text{P}(\text{car accident} \mid \text{females})=\frac{15}{44}=0.3409$ $\text{RR}=\frac{0.5536}{0.3409}$ $=1.624$ The claim is incorrect, as 1.624 is not close to 2.	One risk correctly calculated.	Correct relative risk calculated.	Correct relative risk and its explanation AND Rejected the claim.
(c)	• If conditions are different next month (season/special events/public holidays/Christmas), the table may not be useful for predicting next month's numbers. • Performing an estimation based on just one sample may not give an accurate proportion of fractures occurring in the next month. *Accept any other valid reason.*	State and clearly explained ONE reason.	State and clearly explained TWO reasons.	

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)(i)	$P(1000 < X < 1020)$ $= P(-2 < Z < -2)$ $= 0.9544$	Correct probability.		
(a)(ii)	$P(X > 1016.2)$ $= P(Z > 1.24)$ $= 0.5 - P(0 < Z < 1.24)$ $= 0.5 - 0.3925$ $= 0.1075$	Correct probability.		
(b)	$P(X \le k) = 0.05$ By symmetry $P(X \ge 2020 - k) = 0.05$ $P(0 \le X \le 2020 - k) = 0.45$ $\frac{2020 - k - 1010}{5} = 1.645$ $1010 - k = 1.645 \times 5$ $k = -(1.645 \times 5 - 1010)$ $= 1001.775$ The maximum volume of an underweight bottle of milk is 1001.8 mL.	Correctly formulated the given condition. (line 1 or equivalent)	Correct maximum volume of milk bottle calculated.	
(c)(i)	**Similarity** • Both the normal (bell-shaped) curves for the volume of milk and volume of apple juice are centred about 1010 mL, as both distributions have the same mean. **Difference** • The curve for apple juice is wider, as the volume of apple juice is more variable. • The peak of the curve for milk is higher than the peak of the curve for apple juice.	ONE similarity stated. OR ONE difference stated.	ONE similarity stated. AND ONE difference stated.	

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(d)(i)	$(100-12.5)\times p-12.5\times(1-p)=-11.38$ $87.5p-12.5+12.5p=-11.38$ $100p=1.12$ $p=0.0112$ $(100-12.5)\times p-12.5\times(1-p)=-10.38$ $87.5p-12.5+12.5p=-10.38$ $100p=2.12$ $p=0.0212$ Therefore, the proportion is between 0.0112 and 0.0212.		Correctly calculated either the lower probability or the upper probability.	Correct range of proportions is calculated.
(d)(ii)	Expected value is negative, irrespective of what proportion in the range is used.	Merely stated that the expected value is negative.	Full explanation given.	

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Reasonable start / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

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