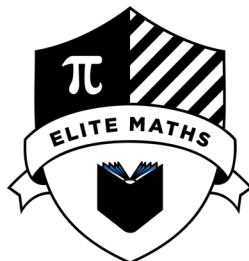


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Level 1 Mathematics and Statistics CAT, 2020 v1

91027 Apply algebraic procedures in solving problems

Wednesday 15 September 2020

Credits: Four

You should attempt ALL the questions in this booklet.

Calculators may NOT be used.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. 'Guess and check' and 'correct answer only' methods do not demonstrate relational thinking and will limit the grade for that part of the question to a maximum of Achievement. Guess and check and correct answer only may only be used a maximum of one time in the paper and will not be used as evidence of solving a problem.

A candidate cannot gain Achievement in this standard without solving at least one problem.

Answers must be given in their simplest algebraic form.

Where a question is given in words you will be expected to write an equation.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement Criteria

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic procedures in solving problems.	Apply algebraic procedures, using relational thinking, in solving problems.	Apply algebraic procedures, using extended abstract thinking, in solving problems.

Overall level of performance

QUESTION ONE

(a) Solve the equation $6x^2 + 7x + 2 = 0$.

(b) Write $\frac{3n+1}{3} - \frac{2n-1}{5}$ as a single fraction.

(c) Find the value of p so that $2^{p^2-2p+8} = \frac{4^{p+1}}{2^{-2}}$.

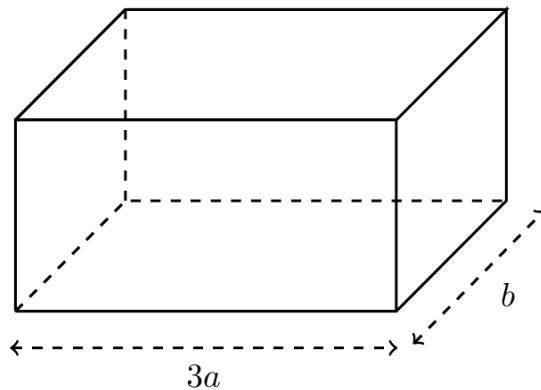
QUESTION TWO

(a) The kinetic energy of an object, E , is given by

$$E = \frac{1}{2}mv^2$$

Find the kinetic energy of a ball with $m = 4$ and $v = 15$.

(b) The diagram below shows a rectangular prism.



*Diagram is
NOT to scale*

The volume of the rectangular prism is $36ab^3 - 27a^2b^2$.

Find the height in terms of a and b .

Remember that *volume of rectangular prism = length \times width \times height*.

Q1	Expected Coverage	Grade (generated by correctly demonstrating the procedures listed in EN4) Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent)
(d) (i)	$2x - 3 > x + 6$ $x > 9$ <p>Accept $9 < x$.</p>	For award of u: • correct inequality.
(d) (ii)	<p>Area of square = $3 \times$ Area of triangle</p> $(2x - 3)^2 = 3 \times \frac{1}{2} \times (x + 6) \times (2x - 3)$ $2(2x - 3)^2 - 3(x + 6)(2x - 3) = 0$ $(2(2x - 3) - 3(x + 6))(2x - 3) = 0$ $(4x - 6 - 3x - 18)(2x - 3) = 0$ $(x - 24)(2x - 3) = 0$ $x = 24$ <p>From part (d)(i), $x = \frac{3}{2}$ is omitted since $x > 9$.</p>	For award of u: ONE of: • forms the correct equation from the question. • expands either side of the equation correctly. • consistently simplifies their equation to a quadratic equation. • consistent factorisation of the quadratic equation into two brackets • consistent solving of the factorised quadratic equation.
		For award of r: • consistent solution(s) from the quadratic equation.
		For award of t: • correct value of x found and the invalid solution is omitted with justification.

Q3	Expected Coverage	<p>Grade (generated by correctly demonstrating the procedures listed in EN4)</p> <p>Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent)</p>
(a)	$2(x-3) - 3(3-2x) < 17$ $2x - 6 - 9 + 6x < 17$ $8x < 32$ $x < 4$ <p>Accept $4 > x$.</p>	<p>For award of u:</p> <p>ONE of:</p> <ul style="list-style-type: none"> • expansion of all brackets. • consistently solved inequality after one correct expansion. • solution given as an equality (instead of an inequality). <p>For award of r:</p> <ul style="list-style-type: none"> • Correct solution.
(b)	$\frac{2x^2 - 10xy}{2} \div \frac{(x-5y)^2}{2y}$ $= x(x-5y) \times \frac{2y}{(x-5y)^2}$ $= \frac{2xy}{x-5y}$	<p>For award of u:</p> <p>ONE of:</p> <ul style="list-style-type: none"> • correct factorisation of the numerator of the left expression. • consistently simplified. <p>For the award of r:</p> <ul style="list-style-type: none"> • correct solution.
(c)	$\frac{C+B}{A^6} \times A^4 = \left(\frac{C}{A^4 B} - \frac{B}{A^4 C} \right) \times A^4$ $\frac{C+B}{A^2} = \frac{C}{B} - \frac{B}{C}$ $\frac{C+B}{A^2} = \frac{C^2 - B^2}{BC}$ $\frac{A^2}{C+B} = \frac{BC}{(C-B)(C+B)}$ $A = \pm \sqrt{\frac{BC}{C-B}}$ <p>The conditions $A \neq 0$, $B \neq 0$, $C \neq 0$, $C+B \neq 0$ and $C-B \neq 0$ are not required.</p>	<p>For award of u:</p> <ul style="list-style-type: none"> • one valid step to isolate A^n term on one side. • performing cross multiplication. <p>For the award of r:</p> <ul style="list-style-type: none"> • consistent simplification of the equation. <p>For the award of t:</p> <ul style="list-style-type: none"> • expression for A (\pm sign not required).

Q3	Expected Coverage	<p>Grade (generated by correctly demonstrating the procedures listed in EN4)</p> <p>Requirements are for the student responses to be correct (ignoring numerical errors) unless the statement specifies consistent)</p>
(d)	<p>Let the three numbers be n, $n+1$, $n+2$.</p> $n + (n+1) + (n+2) = 111$ $3n + 3 = 111$ $3n = 108$ $n = 36$ <p>Therefore, the greatest number is 38.</p>	<p>For award of u:</p> <p>ONE of:</p> <ul style="list-style-type: none"> • expressions for the three numbers. • consistently formed linear equation. • consistent simplification of the equation. • Stated 36 or 37 as the answer. <p>For award of r:</p> <ul style="list-style-type: none"> • correct solution.
(e)	<p>Let H be the height of the cylinder.</p> $\pi R^2 H + \frac{4}{3} \pi R^3 = 9\pi R^3$ $\frac{1}{3} \pi R^2 (3H + 4R) = 9\pi R^3$ $3H + 4R = 27R$ $3H = 23R$ $H = \frac{23}{3} R$ <p>Therefore, the height of the cylinder is $\frac{23}{3}$ times its radius.</p> <p>Accept 7.7 times.</p>	<p>For award of u:</p> <p>ONE of:</p> <ul style="list-style-type: none"> • writes expression for the volume of the cylinder. • writes expression for the volume of the sphere. • writes the sum equation. <p>For the award of r:</p> <ul style="list-style-type: none"> • consistently solves the problem. • simplification is almost correct with a minor error. <p>For the award of t:</p> <ul style="list-style-type: none"> • correct solution.

9 1 0 2 8



Level 1 Mathematics and Statistics, 2020 v1

91028 Investigate relationships between tables, equations and graphs

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Investigate relationships between tables, equations and graphs.	Investigate relationships between tables, equations and graphs, using relational thinking.	Investigate relationships between tables, equations and graphs, using extended abstract thinking.

You should attempt ALL the questions in this booklet.

Show ALL working.

Grids are provided on some pages. This is working space for the drawing of a graph or a diagram, constructing a table, writing an equation, or writing your answer.

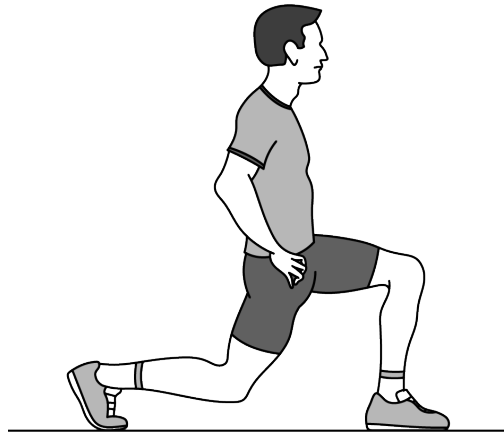
If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

QUESTION ONE

- (a) Tim is training for a cross fit competition.
One of his routine exercises is lunges.



Starting with 10 lunges on day 1, Tim is to do 4 more lunges than he did in the previous day each day.

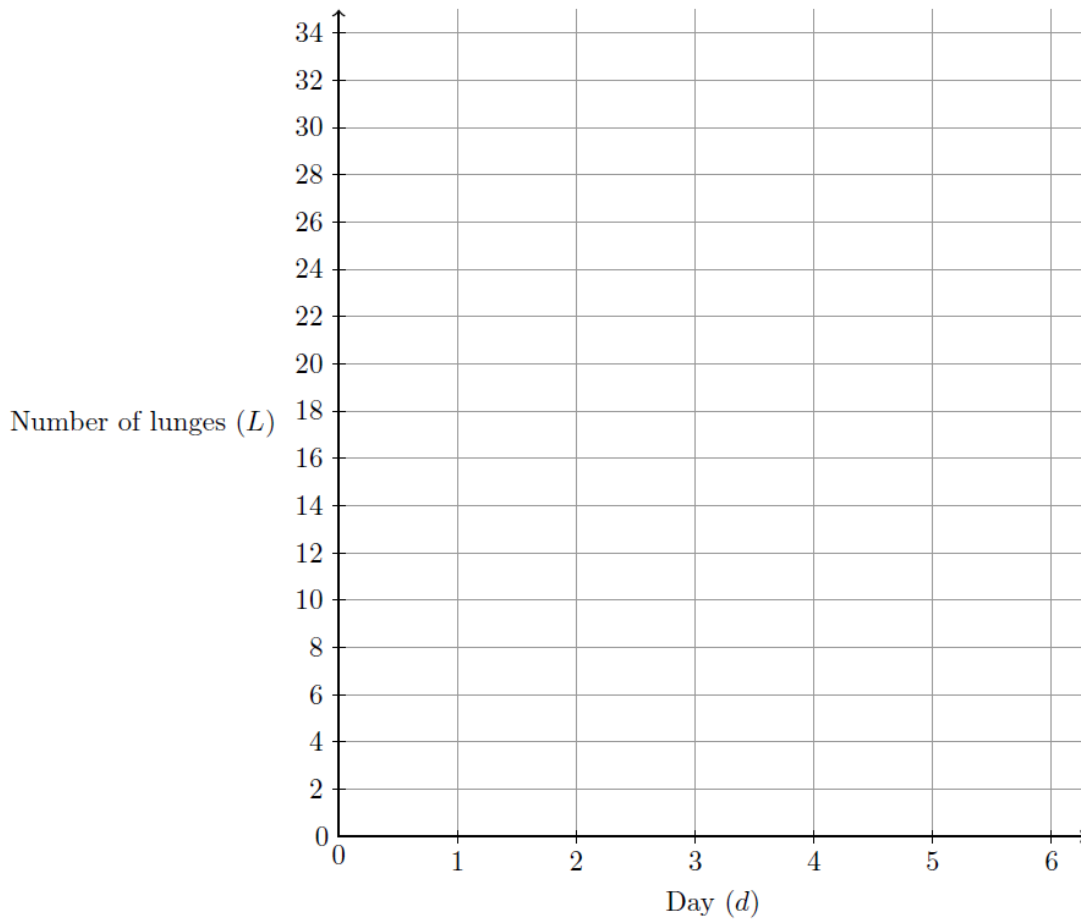
The table below lists how many lunges he is to do for the next 6 days.

Day, d	Number of lunges, L
1	10
2	14
3	18
4	22
5	26
6	30

- (i) Write the equation for the number of lunges done, L , on day d .

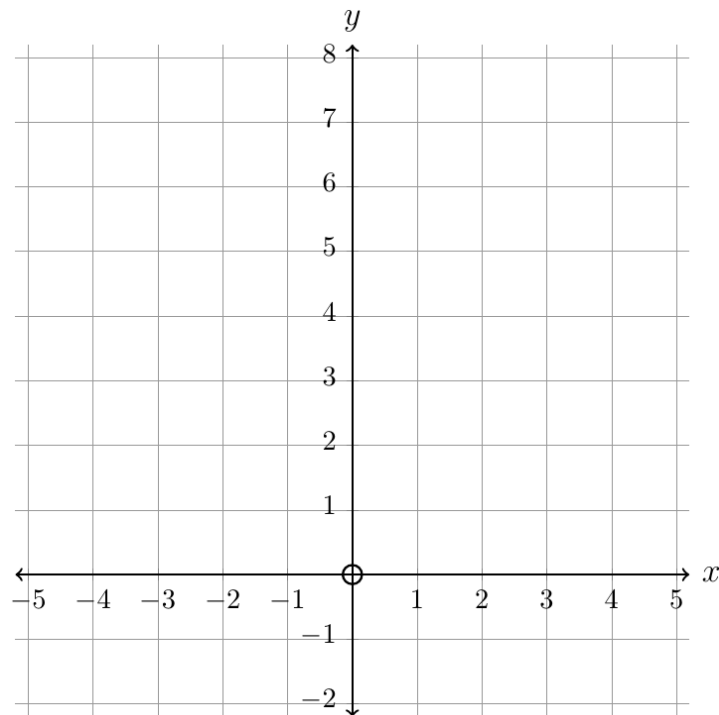
- (ii) Suppose that Tim continues this pattern.
Use the equation in part (i) to find on what day Tim will do 106 lunges.

(iii) On the grid below, sketch the graph of the number of lunges done by Tim each day.



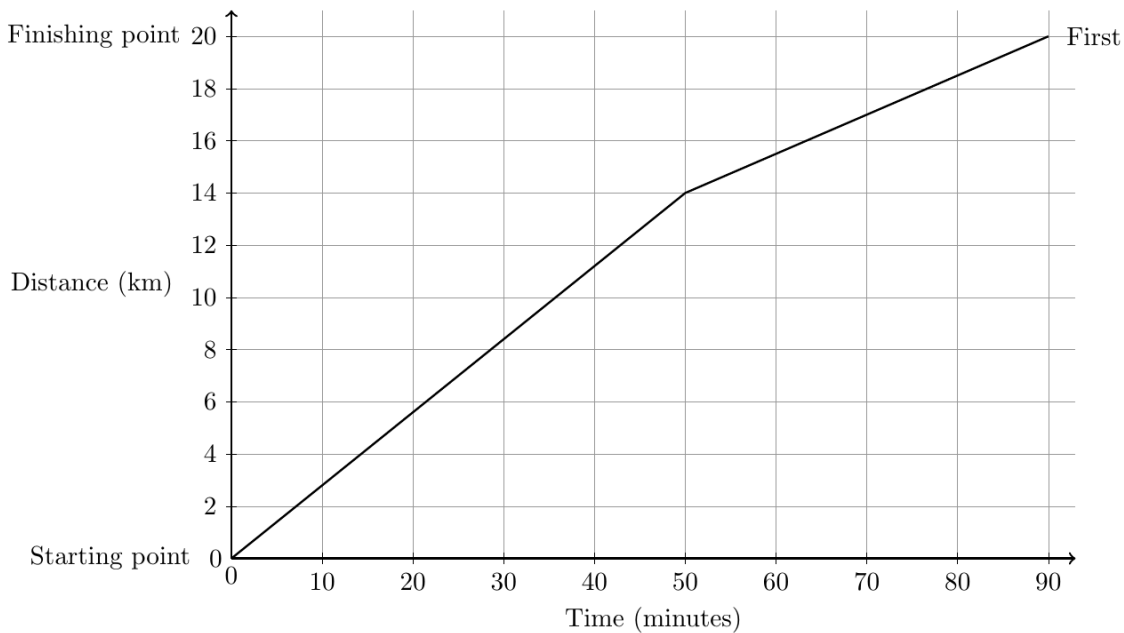
(iv) In total, how many lunges will Tim have done by the end of day 6?

- (b) The equations $y = -2(x+a)(x+b)+8$ and $y = -2(x+1)(x-3)$ represent the same parabola.
Find the values of the constants a and b .
You may use the set of axes provided below if necessary.



QUESTION TWO

- (a) A 20 km run is also part of Tim's training routine.
Tim's first run is shown by the graph below.



A couple of weeks later, Tim attempts his second run.

For the first 30 minutes, Tim runs at a constant speed of $\frac{1}{3}$ km per min.

After having a rest for 10 minutes, he runs the rest of the distance at a constant rate.

After 90 minutes, he is still 2 km from the finishing point.

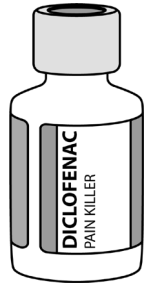
- (i) Give the equation that represents Tim's **first** run for:

1. between 0 minutes and 50 minutes

2. between 50 minutes and 90 minutes

- (ii) Sketch the graph of Tim's **second** run on the same set of axes as the graph of the first run.

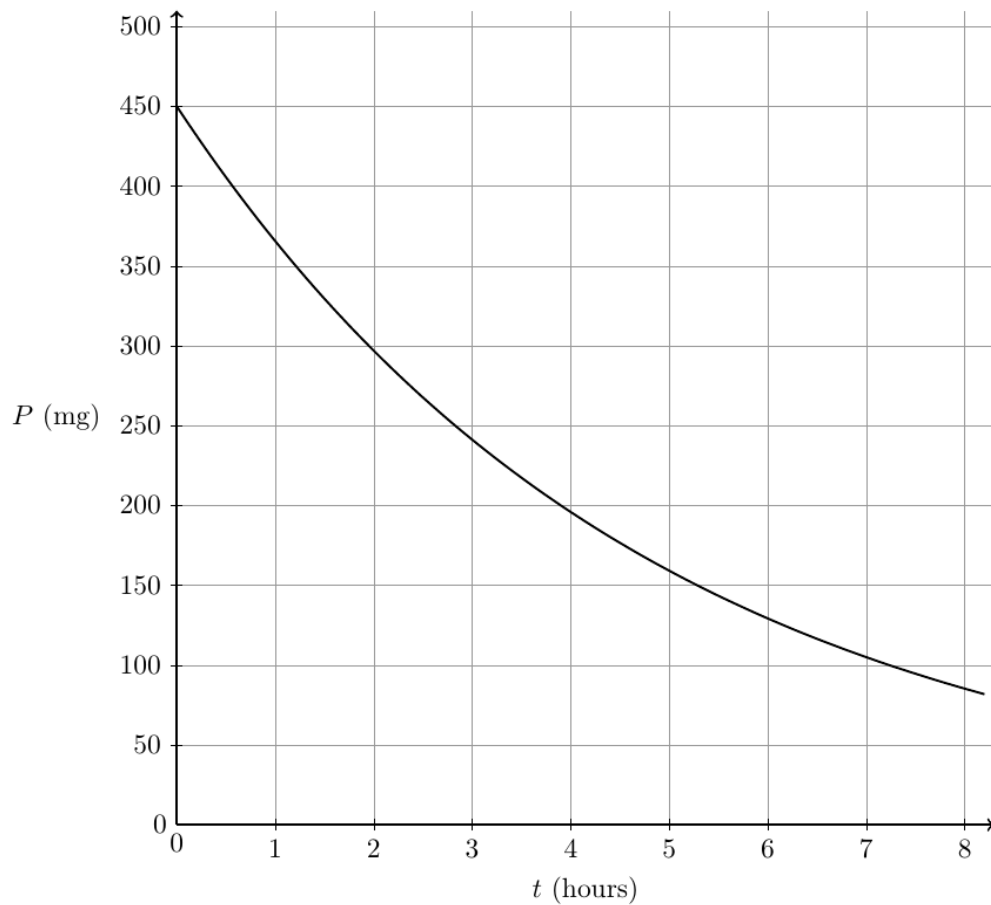
(b)

ASSESSOR'S
USE ONLY

The amount of a painkiller remaining in the bloodstream, measured in P milligrams t hours after it has been injected intravenously* to patients, is given by

$$P = 450 \times 0.5^{0.3t}.$$

The graph below shows P versus t .



(i) How many milligrams of painkiller are injected to patients?

* through a vein

Question TWO	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
(a) (i) 1	$Distance = 0.28 \times Time$ Accept using other variables such as x and y .	Correct equation found.		
(a) (i) 2	$Distance = 0.15 \times Time + 6.5$ Accept using other variables such as x and y .	Either correct slope or correct y -intercept.	Correct equation.	
(a) (ii)	<p>The graph plots Distance (km) against Time (minutes). The y-axis ranges from 0 to 20 km in increments of 2. The x-axis ranges from 0 to 90 minutes in increments of 10. Two lines are shown: 'First' and 'Second'. The 'First' line starts at (0,0) and passes through (50, 14). The 'Second' line starts at (0,0) and passes through (90, 18). The two lines intersect at approximately (35, 9.8).</p>	TWO phases of the runs are shown.	All THREE phases of the run are shown.	
(a) (iii)	<p>Method 1 Equating the expression of the first attempt for $0 \leq Time \leq 50$ ($Distance = 0.28 \times Time$) to $Distance = 10$ gives</p> $0.28 \times Time = 10$ $Time = 35.7 \text{ min}$ <p>Method 2 The two graphs intersect around $Time = 35$. Allow 35 ± 2 for this method.</p>	CAO.	Correct answer with clear explanation.	
(a) (iv)	<p>Tim would have mostly likely been unwell during the second attempt.</p> <p>Tim's first attempt seems well-paced throughout, as he maintained a constant speed for the first 50 minutes (0.28 km per min), and then again maintained a constant speed (although a little slower at 0.15 km per min) for the last 40 minutes.</p> <p style="text-align: right;"><i>continued on the next page</i></p>	Stated that Tim was unwell during his second run.	Fully explained why EITHER Tim's first run shows better performance. OR Tim's second run shows under-performance.	Fully explained why Tim's first run shows better performance. AND Tim's second run shows under-performance.

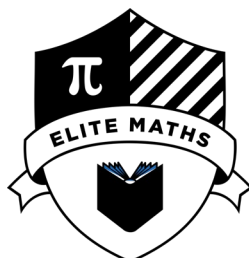
Question THREE	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
(b) (iv)	<p>1. The initial amount of painkiller injected is 450 milligrams which is more than the initial amount of sedative injected which is 220 milligrams.</p> <p>The vertical intercept of the graph $y = P(t)$ is positioned higher (0, 450) than that of the graph of $y = S(t)$ (0, 220).</p> <p>2. The equation $S = 220 \times 0.32^{0.25t}$, the base is 0.32, which is smaller than that for the equation $P = 450 \times 0.5^{0.3t}$, while the coefficients of t are relatively close in value. This means that S has a stronger exponential decay than P.</p> <p>The graph of $y = S(t)$ has a more pronounced exponential decay than the graph of $y = P(t)$.</p> <p>3. As t increases, both P and S decrease exponentially to approach 0.</p> <p>Both of the graphs have a horizontal asymptote at $y = 0$.</p>	ONE item fully compared.	TWO items fully compared.	THREE items fully compared.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Attempt at ONE question.	1 of u OR partial solution in TWO questions.	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Scores

	Not Achieved	Achievement	Achievement with merit	Achievement with Excellence
Score range	0 – 7	8 – 14	15 – 20	21 – 24

9 1 0 3 1



Level 1 Mathematics and Statistics, 2020 v1

91031 Apply geometric reasoning in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply geometric reasoning in solving problems.	Apply geometric reasoning, using relational thinking, in solving problems.	Apply geometric reasoning, using extended abstract thinking, in solving problems.

You should attempt **ALL** the questions in this booklet.

Show **ALL** working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

TRUSSES

Trusses are used in numerous construction projects such as airport terminals, aircraft hangers and leisure buildings.

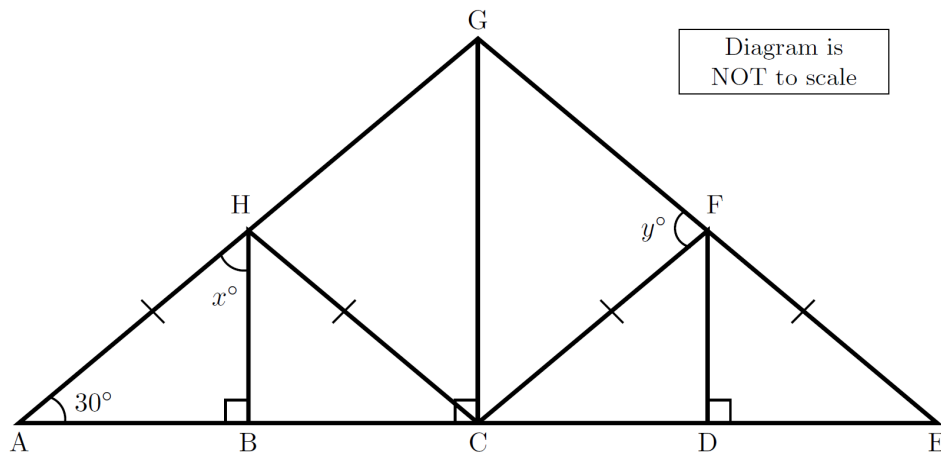
QUESTION ONE

(a) The following diagram shows a Howe truss, made up of steel beams.

BH, CG, and DF are perpendicular to AE.

AH = CH = CF = EF and BH = DF.

Angle BAH = 30° .



(i) Show that the size, x , of angle BHA is 60° .

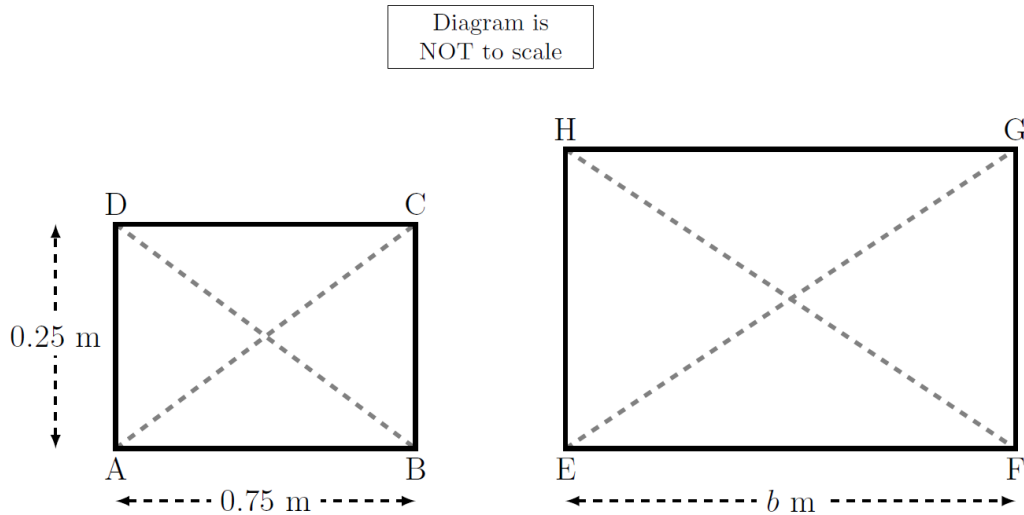
Justify your answer with clear geometrical reasoning.

(ii) Find the size, y , of angle CFG.

Justify your answer with clear geometrical reasoning.

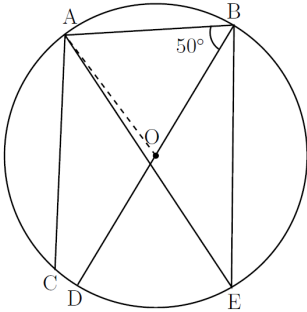
QUESTION TWO

- (a) The diagram on the right shows one of the rectangular supports that Howe trusses are placed on. The diagram on the left shows a miniature model of the rectangular support. The scale factor is 32.

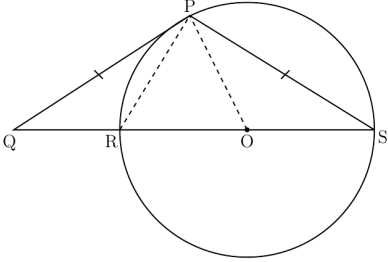


- (i) Calculate the value of b .

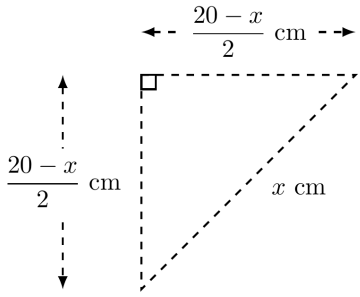
- (ii) Calculate the perimeter of three of the actual rectangular supports put together side by side, without any gaps in between.
Show your working clearly.

Question ONE	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
(d)	<p>A line connecting point O and point A has been drawn, as shown in the diagram below (dotted line).</p>  <p>Consider the isosceles triangle OAB.</p> $\angle BAO = 50^\circ$ <p>(base \angle s isos Δ =)</p> <p>Therefore $\angle AOB = 180^\circ - 50^\circ - 50^\circ = 80^\circ$. ($\angle$ s in Δ add to 180°)</p> $\begin{aligned} \angle AEB &= \frac{\angle AOB}{2} \\ &= \frac{80^\circ}{2} \\ &= 40^\circ \end{aligned}$ <p>(\angle centre = $2 \times \angle$ at circum.)</p> <p>Since chord AC and chord BE are parallel $\angle CAE = \angle AEB$ $= 40^\circ$ (alt \angle s //)</p> <p><i>Accept any other valid proof.</i></p>	Correct $\angle BAO$ or correct $\angle AOB$ with justification.	Correct $\angle AEB$ found with justification.	Clear working shown to obtain the correct $\angle CAE$.

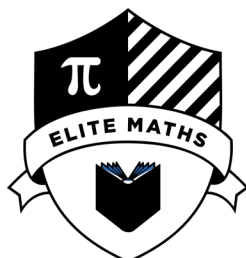
NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Attempt at ONE question.	1 of u OR partial solution in TWO questions.	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Question TWO	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
(c)	<p>Note the dotted lines PR and OP are drawn in the diagram below.</p>  <p>Since PQS and OPS are isosceles triangles $\angle PQS = \angle PSQ = \angle SPO$.</p> <p>Since an exterior angle of a triangle is equal to the sum of the opposite interior angles, $\angle POQ = 2 \times \angle PQS$.</p> <p>Since PQ is a tangent to the circle, PQ is perpendicular to OP. This means that OPQ is a right-angled triangle.</p> <p>Since the sum of the internal angles of a triangle is 180° $2 \times \angle PQS + \angle PQS + 90 = 180$ $3\angle PQS = 90$ $\angle PQS = 30$</p> <p>This means that $\angle POQ = 60^\circ$.</p> <p><i>Accept any other valid proof.</i></p>	<p>Stated that PQS is an isosceles triangle.</p> <p>OR</p> <p>Stated that OPS is an isosceles triangle.</p> <p>OR</p> <p>Wrote down any other statement apart from above two that is necessary to find the correct answer.</p>	<p>Stated that angle OPQ is a right angle.</p> <p>AND</p> <p>Correctly calculated any one of the three angles correctly (PQS, PSQ or SPO).</p>	<p>Correct answer with every step justified.</p>

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Attempt at ONE question.	1 of u OR partial solution in TWO questions.	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Question THREE	Evidence	Achievement	Achievement with Merit	Achievement with Excellence
(a)	<p>BN = 4.5 and NQ = 6.</p> <p>By Pythagoras' theorem</p> $BQ = \sqrt{4.5^2 + 6^2}$ $= 7.5 \text{ m}$	Correct answer.		
(b)	<p>$\angle FGI = 129^\circ$ (corres \angle s)</p> <p>$\angle GFI = 31^\circ$ (vert opp \angle s =)</p> $129 + 31 + x = 180$ $x = 180 - 160$ $= 20$ <p>(\angle s in Δ add to 180°)</p>	One relevant angle found with justification.	Correct angle found with every step justified.	
(c)	<p>Let the side length of the regular octagon be x. The diagram below shows a cut-out right-angled triangle.</p>  <p>Using Pythagoras' theorem</p> $\left(\frac{20-x}{2}\right)^2 + \left(\frac{20-x}{2}\right)^2 = x^2$ $2\left(\frac{20-x}{2}\right)^2 = x^2$ $1.414\dots\left(\frac{20-x}{2}\right) = x$ $20-x = (1.414\dots)x$ $x \approx 8.284 \text{ cm}$ <p>Accept using GC.</p>		Formed a correct equation using Pythagoras' theorem.	<p>Correct length.</p> <p>Accept any rounding.</p>

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Level 1 Mathematics and Statistics, 2020 v1

91037 Demonstrate understanding of chance and data

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Demonstrate understanding of chance and data.	Demonstrate understanding of chance and data, justifying statements and findings.	Demonstrate understanding of chance and data, showing statistical insight.

You should attempt **ALL** the questions in this booklet.

Show **ALL** working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

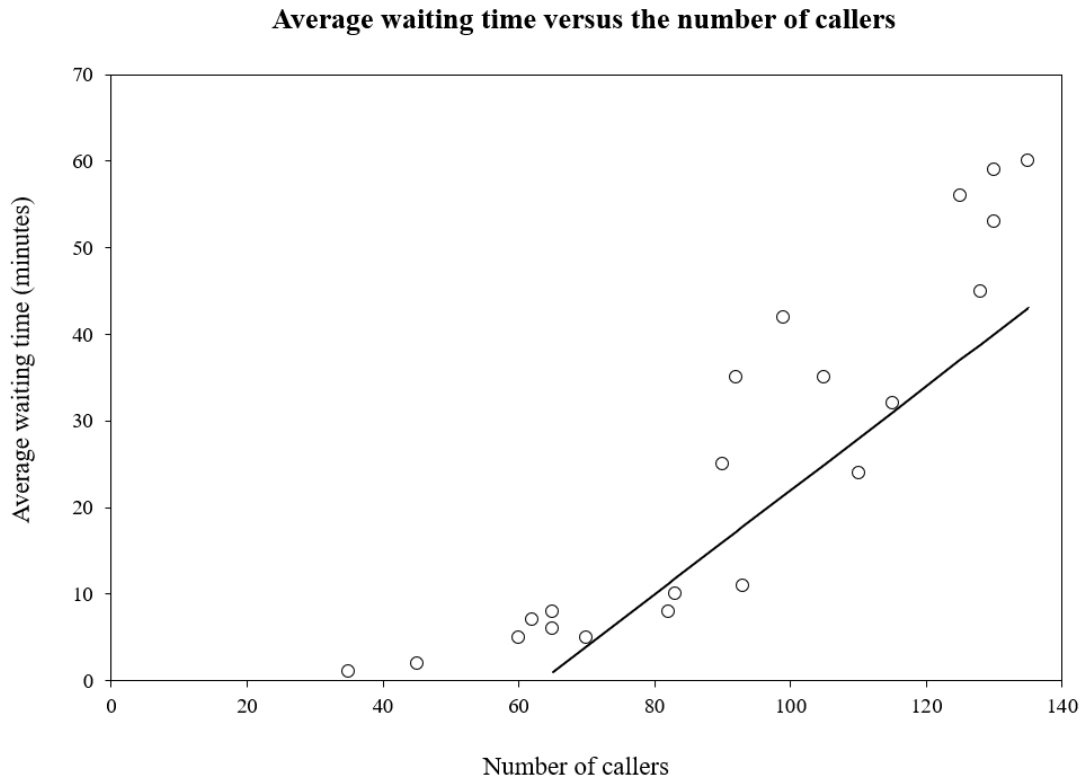
YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

QUESTION ONE

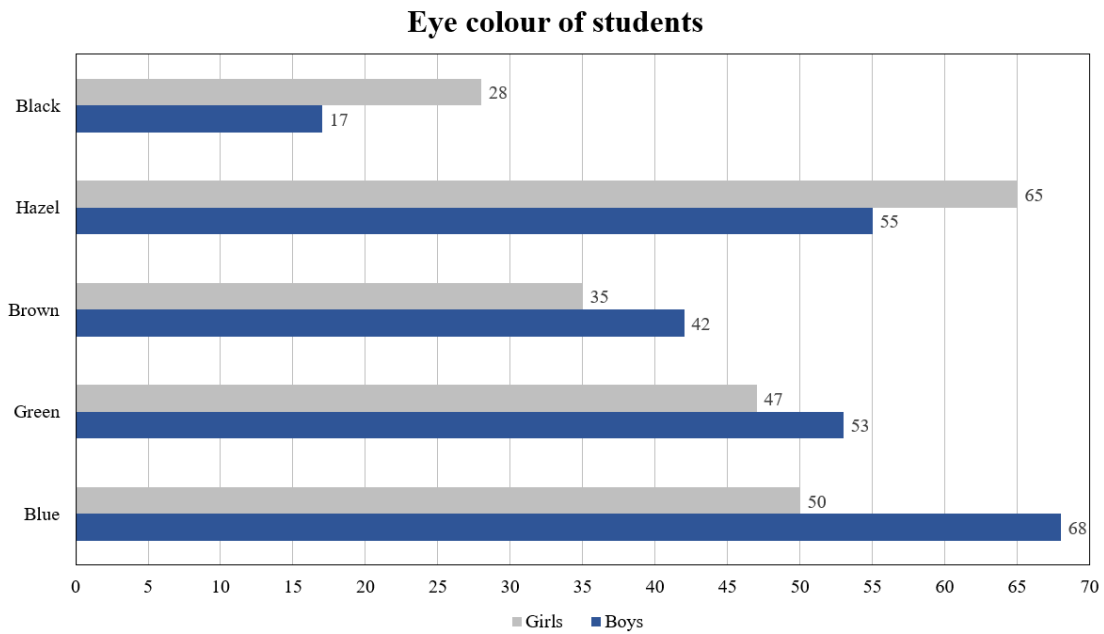
- (a) When there is a large number of callers at a travel agency, some callers are put on hold. The graph shows the average waiting time (in minutes) for callers when they are put on hold on selected days, and the number of calls received on that day.

A line that approximates the relationship between the average waiting time and the number of callers has also been drawn on the graph.



- (i) Find the lower quartile of the average waiting time.
Justify your answer.

- (b) The eye colours of the 460 students who currently attend John Paul College were recorded. The following graph displays the results.



- (i) What is the total number of boys that currently attend John Paul College?
Show your working clearly.

- (b) The graph below shows the number of wine bottles sold at a liquor shop, during each quarter between 2015 and 2020.



- (i) During which quarter does the liquor shop usually sell the highest number of bottles?
Justify your answer.

QUESTION THREE

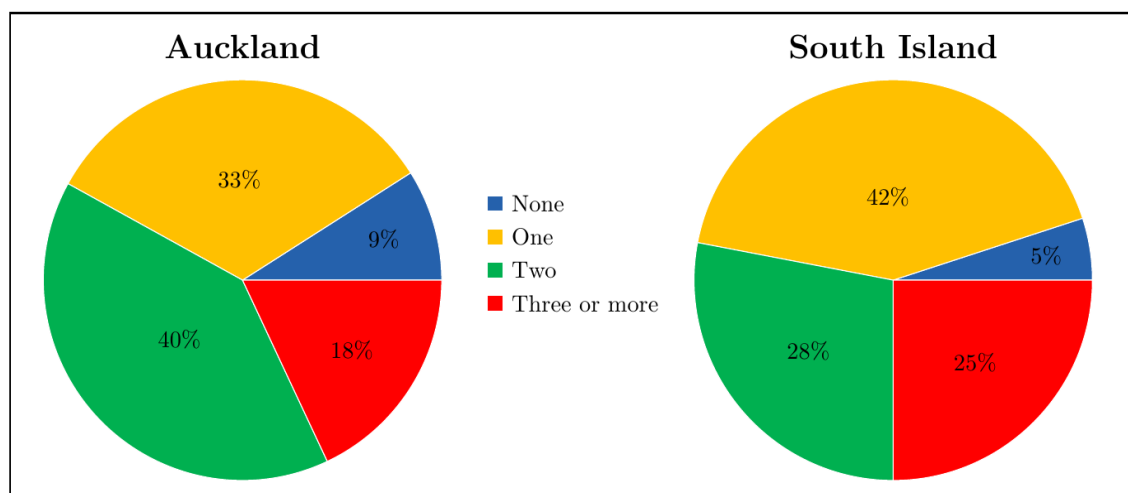
- (a) The number of motor vehicles that households in Auckland own are shown side-by-side with those for the South Island below.

All figures are for households in occupied private dwellings.

Number of motor vehicles	Auckland	South Island
None	132,102	57,773
One	484,374	485,289
Two	587,121	323,526
Three or more	264,204	288,863
Total	1,467,801	1,155,451

Source: Statistics New Zealand, modified

The percentages of each category for both Auckland and the South Island households are displayed in the graph below.



- (i) Use the data shown in the table to show that the percentage of households in Auckland owning two motor vehicles is 40%.

Show all working.

TWO	Expected Coverage	Achievement	Achievement with Merit	Achievement with Excellence
(a) (i)	Reasonable answers such as due to reverse psychology/curiosity.	Correct answer.		
(a) (ii)	$\frac{35}{78} \times \frac{14}{52} + \frac{43}{78} \times \frac{38}{52}$ $= \frac{177}{338} \text{ or } 0.5237$	At least one correct probability shown in the working.	Correct answer with working shown.	
(a) (iii)	<p>Expected value</p> $= 60 \times \frac{35+38}{78+52}$ ≈ 33.69 <p>Since the proportions of the people that pressed the button in Group 1 ($35/78 \approx 0.4487$) and Group 2 ($38/52 \approx 0.7308$) vary, Group 1 and Group 2 could be of different demographics. The last group could also be of a different demographic.</p> <p>The arrangement (lighting, sound and visual effects) of the experiment could have changed prior to when the last group walks past the panel.</p>	Correct expected value.	Correct expected value. AND ONE valid statement.	Correct expected value. AND TWO valid statements.
(b) (i)	For the last 5 years, the highest number of bottles were sold during the fourth quarter (Q4).	Correct answer with justification.		
(b) (ii)	<p>It smooths the data so that the graph is less affected by outliers/extreme values.</p> <p>It shows long term trends.</p> <p>It helps by reducing any seasonal variations</p> <p>It helps with reading the time series graph better.</p> <p>It helps to make predictions.</p>	ONE valid statement.	TWO valid statements.	THREE valid statements.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

THREE	Expected Coverage	Achievement	Achievement with Merit	Achievement with Excellence
(b) (i)	$\begin{aligned} \text{IQR} &= uq - lq \\ &= 775 - 351 \\ &= 424 \end{aligned}$ <p>424,000</p>	CAO		
(b) (ii)	<p>1. Shape Similar since both boxplots seem reasonably unimodal</p> <p>2. Symmetry/skew Boxplot for comedy is skewed and boxplot for fairy tale also seems skewed.</p> <p>3. Shift The middle 50% box of comedy has been visibly shifted to the right of the 50% box of fairy tale.</p> <p>5. Centre The mean/median of comedy is greater than the mean/median of fairy tale.</p> <p>6. Spread The box plots show that the variation of comedy (IQR=424) is significantly greater than that for fairy tale (IQR=127). The differences in standard deviations (225 versus 79) and range (906 versus 339) further supports this.</p>	ONE valid comparison statement about ONE significant feature.	TWO valid comparison statements about TWO significant features.	THREE valid comparison statements about THREE significant features.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Cut Scores

	Not Achieved	Achievement	Achievement with merit	Achievement with Excellence
Score range	0 – 7	8 – 14	15 – 20	21 – 24